9. The section of a crane hook is trapezoidal, whose inner and outer sides are 90 mm and 25 mm respectively and has a depth of 116 mm. The center of curvature of the section is at a distance of 65 mm from the inner side of the section and load line passes through the center of curvature. Find the maximum load the hook can carry, if the maximum stress is not to exceed 70 MPa. **VTU, Dec. 07/Jan. 08**

10. a) Differentiate between a straight beam and a curved beam with stress distribution in each of the beam.

    b) Fig. 1.39 shows a 100 kN crane hook with a trapezoidal section. Determine stress in the outer, inner, Cg and also at the neutral fibre and draw the stress distribution across the section AB. **VTU, Jun/July. 08**

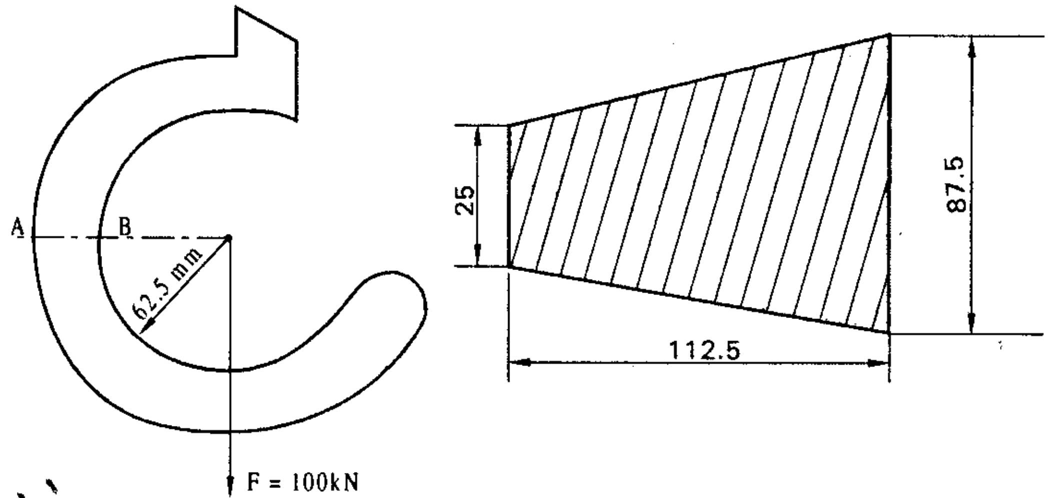


Fig.1.39

11. A closed ring is made up of 50 mm diameter steel bar having allowable tensile stress of 200 MPa. The inner diameter of the ring is 100 mm. For load of 30 kN find the maximum stress in the bar and specify the location. If the ring is cut as shown in part -B of Fig. 1.40, check whether it is safe to support the applied load. **VTU, Dec. 08/Jan. 09**

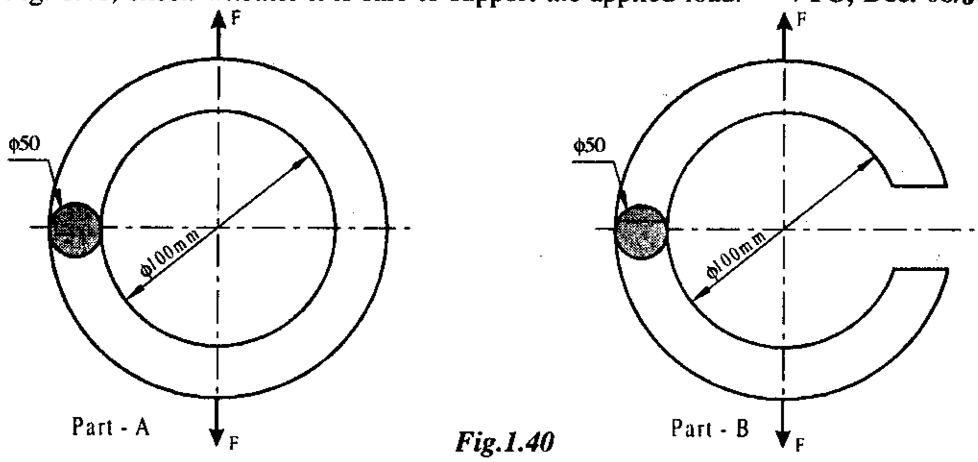


Fig.1.40

UNIT 2

CYLINDERS AND CYLINDER HEADS

2.1 INTRODUCTION

Cylindrical pressure vessels are classified into two groups. (i) Thin cylinders and (ii) Thick cylinders. A cylinder is considered to be thin when the ratio of its wall thickness to the internal radius is less than $1/10$. In thin cylinder the stress distribution is assumed to be uniform over the thickness of wall.

2.2 STRESSES IN A THIN CYLINDER

When a thin cylinder is subjected to internal pressure, its walls are subjected to two types of tensile stresses.

(i) Circumferential stress or Hoop stress or Tangential stress

(ii) Longitudinal stress

The stress acting along the circumference of the cylinder is called circumferential stress and the stress acting along the length of the cylinder is called longitudinal stress. Circumferential stress is also called as hoop stress. The stress set up in two troughs is circumferential stress and the stress set up in two cylinders is longitudinal stress.

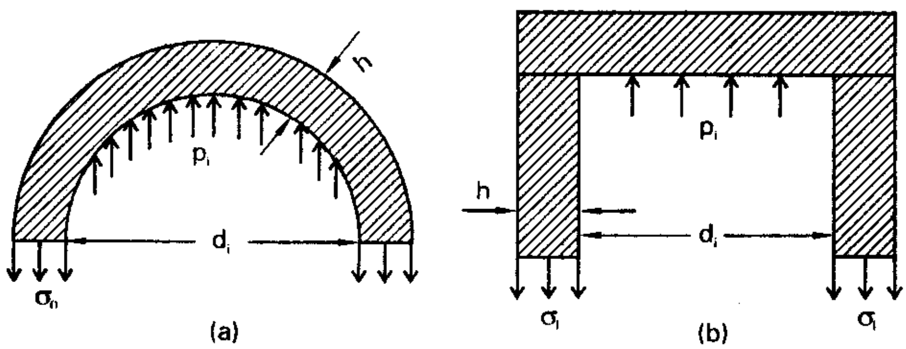


Fig. 2.1

Considering the equilibrium of forces in the circumferential direction [Fig. 2.1 (a)].

$$d_i\, p_i\, l = 2\, \sigma_\theta\, .\, hl$$

$\therefore$ Circumferential or Tangential stress $\sigma_\theta = \dfrac{p_i d_i}{2h}$ 2.1

Considering the equilibrium of forces in the longitudinal direction [Fig. 2.1 (b)].

$$p_i \left(\frac{\pi}{4} di^2\right) = \sigma_l\, (\pi d_i h)$$

$\therefore$ Longitudinal stress $\sigma_l = \dfrac{p_i d_i}{4h}$ 2.2

From equations 2.1 and 2.2, it is seen that the circumferential stress σ_θ is twice the longitudinal stress.

$\therefore$ Thickness of thin cylinder wall $h = \dfrac{p_i d_i}{2\sigma_\theta}$ where σ_θ is the permissible tensile stress.

When there is a joint or seam in the cylinder, the efficiency of the joint should be taken in to account.

$$\therefore \sigma_\theta = \frac{p_i d_i}{2h\eta} \quad ..... 2.3$$

$$\sigma_l = \frac{p_i d_i}{4h\eta} \quad ..... 2.4$$

$$\text{and } h = \frac{p_i d_i}{2\sigma_\theta \eta} \quad ..... 2.5$$

where p_i = Internal pressure in N/mm²

d_i = Internal diameter of cylinder in mm

h = Cylinder wall thickness in mm

η = Efficiency of the joint

σ_θ = Circumferential or Tangential stress

σ_l = Longitudinal stress.

2.3 STRESS IN A THIN SPHERICAL VESSEL

A spherical pressure vessel with a thin wall is shown in Fig. 2.2

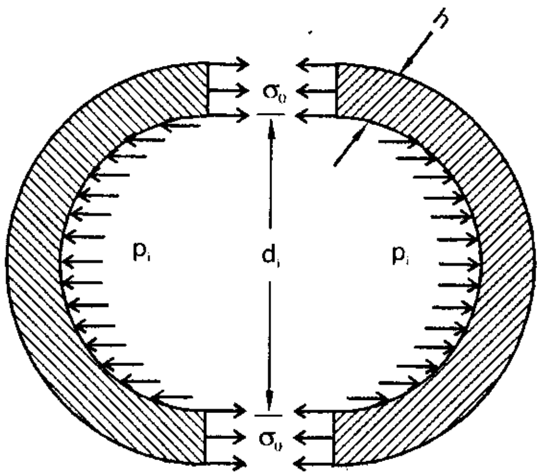


Fig. 2.2

Considering equilibrium of forces

$$\frac{\pi}{4} d_i^2 \, p_i = (\pi \, d_i \, h) \, 6_\theta$$

$$\text{Thickness of shell } h = \frac{p_i d_i}{4\sigma_\theta} \quad \text{..... 2.6}$$

$$\text{Volume of shell } v = \frac{\pi}{6} d_i^3 \quad \text{..... 2.7}$$

When there is a joint, efficiency of the joint should be taken into account.

$$\therefore \text{ Thickness of shell } h = \frac{p_i d_i}{4\sigma_\theta . \eta} \quad \text{.... 2.8}$$

2.4 CHANGE IN DIMENSIONS OF A THIN CYLINDER DUE TO AN INTERNAL PRESSURE

$$\text{Increase in diameter } \delta_d = \frac{p_i d_i^2}{2hE}\left(1 - \frac{\upsilon}{2}\right) \quad \text{.... 2.9}$$

$$\text{Increase in length } \delta_l = \frac{p_i d_i l}{2hE}\left(\frac{1}{2} - \upsilon\right) \quad \text{.... 2.10}$$

$$\text{Increase in volume } \delta_v = \frac{\pi}{4}(d^2 \, \delta_l + 2dl \, \delta_d)\left(1 - \frac{\upsilon}{2}\right)$$ where p_i = Internal pressure

.... 2.11

d_i = Internal diameter of cylinder

h = Thickness of cylinder wall

E = Young's modulus for the material of the cylindrical shell

υ = Poisson's ratio

l = Length of cylinder

δ_d = Change in diameter

δ_l = Change in length

δ_v = Change in volume

v = Original volume

2.5 CHANGE IN DIMENSIONS OF A THIN SPHERICAL VESSEL DUE TO AN INTERNAL PRESSURE

Increase in diameter of a thin spherical vessel $\delta_d = \frac{p_i d_i^2}{4hE}(1-\upsilon)$ 2.12

Increase in volume of a thin spherical vessel $\delta_v = \frac{\pi p_i d_i^4}{8hE}(1-\upsilon)$ 2.13

Example : 1.1

The storage capacity of a seamless cylinder is 0.0515m^3 and it is subjected to an internal pressure of 15 MPa. The cylinder material is alloy steel (σ_u = 500 N/mm^2) and a factor of safty 2.5 is used. The length of the cylinder is twice its internal diameter. Determine the thickness of the cylinder wall.

Data :

p_i = 15 MPa = 15 N/mm^2; v = 0.0515 m^3

σ_u = 500 N/mm^2 ; FOS = 2.5 ; $l = 2\,d_i$

Solution :

$$\text{Volume of the cylinder } v = \frac{\pi}{4} d_i^2\, l$$

$$\text{ie., } 0.0515 = \frac{\pi}{4} d_i^2 \times 2d_i$$

$\therefore$ Internal diameter of the cylinder di = 0.320 m = 320 mm

Length of cylinder l = 2 × 320 = 640 mm

$$\text{Allowable stress } \sigma_\theta = \frac{\sigma_u}{FOS} = \frac{500}{2.5} = 200 \text{ N/mm}^2$$

$$\therefore \text{ Thickness of cylinder wall } h = \frac{p_i d_i}{2\sigma_\theta} = \frac{15 \times 320}{2 \times 200} = 12 \text{ mm}$$

[Neglecting corrosion allowance and joint efficiency]

Example : 1.2

A hydraulic control for a straight line motion utilizes a spherical pressure tank 'P' which is connected to a working cylinder 'C' as shown in Fig. 2.3. The tank pressure 2.5 N/mm² is maintained by a pump. Determine,

(i) Thickness of the pressure tank plates if its internal diameter is 1m and the allowable tensile stress in the plate material is equal to 62.5 N/mm². The tank is welded with joints having strength equal to that of the plate.

(ii) Internal diameter of cast iron cylinder and its thickness to produce an operating force of 20 kN. Assume a pressure drop of 0.15 N/mm² between the tank and the cylinder and an allowance of 10% of operating force for friction in the cylinder and packing. The allowable stress for the cylinder material is 23.5 N/mm².

(iii) Output power of the cylinder, if the stroke of the piston is 480 mm and the time required for the working stroke is 6 seconds.

(iv) Power of the motor, if the working cycle repeats once in every 36 seconds and the overall efficiency of the hydraulic control is 75% and that of the pump is 65%.

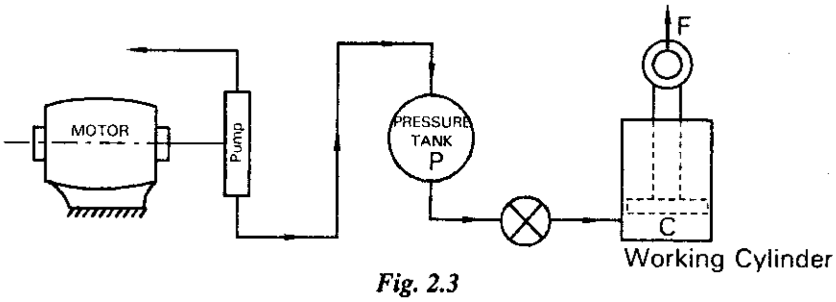


Fig. 2.3

Solution :

(i) Thickness of spherical pressure tank

$p_i = 2.5\ N/mm^2 ; \sigma_0 = 62.5\ N/mm^2 ; \eta = 100\% ; d_i = 1m = 1000\ mm$

Thickness of spherical pressure tank $h = \frac{p_i d_i}{4\sigma_\theta \eta}$ (Neglecting Corrosion allowance)

$$= \frac{2.5 \times 1000}{4 \times 62.5 \times 1} = 10\ mm$$

(ii) Internal diameter of cast iron cylinder and its thickness

Operating force F = 20 kN = 20,000 N

Total force to be produced by the piston = Operating force + 10% F = $20,000 + \frac{10}{100} \times 20000 = 22000N$

Pressure in the cylinder $p_{i_c} = 2.5 - 0.15 = 2.35\ N/mm^2$

Total force produced by the piston = $\frac{\pi}{4} \times d_{i_c}^2 \times p_{i_c}$

$$\text{ie., } 22000 = \frac{\pi}{4} \times d_{i_c}^2 \times 2.35$$

$\therefore$ Internal diameter of cylinder d_{i_c} = 109.2 mm ≅ 110 mm

Thickness of cylinder wall $h_c = \frac{p_{i_c} \times d_{i_c}}{2\sigma_{0_c}} = \frac{2.35 \times 110}{2 \times 23.5}$ = 5.5 mm (Neglecting corrosion allowance)

(iii) Output power of cylinder

Net operating force produced by the piston = 20 kN

Stroke of piston = 480 mm = 0.48 m

Time required for one working stroke = 6 seconds

$\therefore$ Distance moved by the piston per second = $\frac{0.48}{6}$ = 0.08 m

Work done per second = Force × Distance travelled per second = 20 × 0.08 = 1.6 kNm

$\therefore$ Out put power of the cylinder = 1.6 kW

(iv) Power of the motor

The motor is to provide a force of 22 kN to the cylinder

$$\therefore \text{Power of motor} = 22 \times \frac{0.48}{6} \times \frac{1}{0.75} \times \frac{1}{0.65} \times \frac{6}{36} = 0.6017 \text{ kW}$$

Example : 2.3

A cylindrical vessel whose ends are closed by means of hemispherical covers is subjected to an internal pressure of 6 N/mm². The length of the cylindrical portion is twice that of its internal diameter. The allowable tensile stress of the material of the vessel is 82.5 N/mm² and its storage capacity is 0.345 m³. Neglecting the effect of welded joints and the allowance for corrosion, determine its dimensions.

Data :

$$l = 2d_i; \quad \sigma_\theta = 82.5 \text{ N/mm}^2; \quad v = 0.345 \text{ m}^3; \quad p_i = 6 \text{ N/mm}^2$$

Solution :

Volume of the vessel = Volume of the cylindrical portion + Volume of the spherical portion

$$\text{ie., } v = \frac{\pi}{4} d_i^2 \, l + \frac{\pi}{6} d_i^3 = \frac{\pi}{4} d_i^2 (2d_i) + \frac{\pi}{6} pd_i^3$$

$$= \frac{\pi}{2} \cdot d_i^3 + \frac{\pi}{6} d_i^3 = \frac{2}{3} \pi d_i^3$$

$$\therefore \text{ Internal diameter } d_i = \sqrt[3]{\frac{3v}{2\pi}} = \sqrt[3]{\frac{3\times 0.345}{2\pi}} = 0.5482 \text{ m}$$

$$\text{Take, Internal diameter } d_i = 0.55 \text{ m} = 550 \text{ mm}$$

$$\therefore \text{ Length of cylinder } l = 2d_i = 2 \times 550 = 1100 \text{ mm}$$

$$\text{Thickness of cylinder wall h} = \frac{p_i d_i}{2\sigma_\theta} = \frac{6\times 550}{2\times 82.5} = 20 \text{ mm}$$

$$\text{Thickness of spherical end h} = \frac{p_i d_i}{4\sigma_\theta} = \frac{6\times 550}{6\times 82.5} = 10 \text{ mm}$$

∴ Permissible thickness of the vessel h = 20 mm

2.6 THICK CYLINDERS

If the ratio of thickness to internal radius of cylinder is more than $1/10$, then the cylinder is known as thick cylinder. In thick cylinder hoop stress distribution over the thickness of wall is not uniform. It is maximum at the inner circumference and minimum at the outer circumference. Thick cylinders are used to withstand high pressures.

2.7 LAME'S THEORY

The analysis of thick cylinder is complex and hence it is solved by using the following assumptions.

(i) The material of the cylinder is homogeneous and isotropic.

(ii) Plane sections of the cylinder, perpendicular to the longitudinal axis, remain plane under the pressure i.e., longitudinal strain is constant and is independent of radius.

The theory derived after making the above mentioned assumptions is popularly known as Lame's theory.

2.8 STRESSES IN A THICK CYLINDER

Consider a thick cylinder subjected to an internal pressure as shown in Fig. 2.4

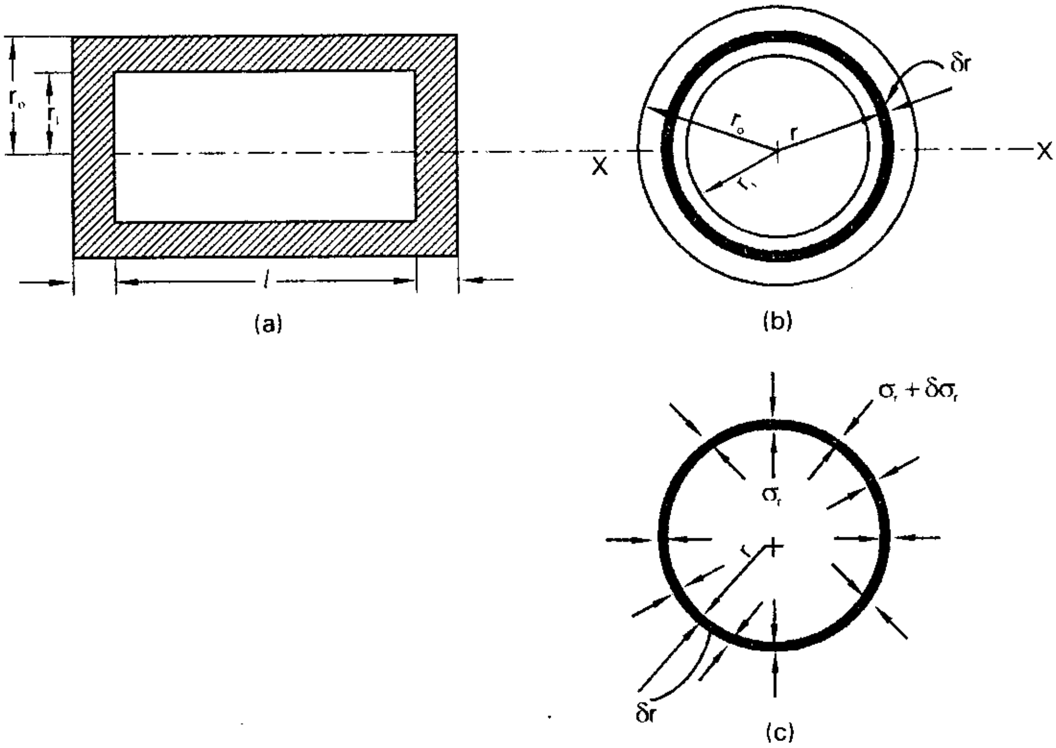


Fig. 2.4

Let r_o = Outer radius of cylinder

r_i = Inner radius of the cylinder

l = Length of cylinder

Consider an elemental ring of thickness δr and radius r of the cylinder as shown in Fig. 2.4 (b) and Fig. 2.4 (c).

Let σ_r = Radial stress on the inner surface of the ring

$\sigma_r + \delta\sigma_r$ = Radial stress on the outer surface of the ring.

σ_0 = Tangential stress induced on the ring

Bursting force on the longitudinal section x – x

$$= \sigma_r (2rl) - (\sigma_r + \delta\sigma_r) 2 (r + \delta r)\, l$$

$$= 2\sigma_r rl - 2\sigma_r.rl - 2\sigma_r \delta r l - 2r\delta\sigma_r.l - 2\delta\sigma_r \delta r.l$$

Neglect $\delta\sigma_r . \delta_r$ as the quantity is very small

$\therefore$ Bursting force on the longitudinal section X – X = $-2\sigma_r \delta r l - 2rl\delta\sigma_r$ .... (i)

Resisting force = Hoop stress × Area on which it acts = $\sigma_0 . 2\delta r.l$ .... (ii)

For equilibrium

$$\text{Bursting force} = \text{Resisting force}$$

$$\text{ie., } -2\,\sigma_r \delta r.l - 2rl\,\delta\sigma_r = \sigma_\theta 2\delta r l$$

$$\text{ie., } -\sigma_r\,\delta r - r\delta\sigma_r = \sigma_\theta \delta r$$

$$\therefore \sigma_\theta = -\sigma_r - r\frac{\delta\sigma_r}{\delta r} \quad \text{.... (iii)}$$

From Lame's theory the longitudinal strain at any point in the section is constant and is independent of the radius. Since the longitudinal strain is constant, the longitudinal stress σ_l will also be constant, Hence at any point at a distance r from the centre, the three principal stresses acting are,

(i) Radial Compressive stress 'σ_r'

(ii) Hoop or Tangential tensile stress 'σ_θ'

(iii) Longitudinal tensile stress 'σ_l'

$\therefore$ Longitudinal strain at this point $\epsilon_l = \dfrac{\sigma_l}{E} - \dfrac{\sigma_\theta}{mE} + \dfrac{\sigma_r}{mE}$

As longitudinal strain is constant, $\dfrac{\sigma_l}{E} - \dfrac{\sigma_\theta}{mE} + \dfrac{\sigma_r}{mE} = \text{constant}$

But σ_l, E and m are also constant

$$\therefore \sigma_\theta - \sigma_r = \text{Constant}$$

$$\text{Take } \sigma_\theta - \sigma_r = 2a$$

$$\therefore \sigma_\theta = \sigma_r + 2a \quad \text{.... (iv)}$$

Equating the equations (iii) and (iv)

$$-\sigma_r - r\frac{\delta\sigma_r}{\delta r} = \sigma_r + 2a$$

$$\text{ie., } \frac{r\delta\sigma_r}{\delta r} = -2(\sigma_r + a)$$

$$\text{ie., } \frac{\delta\sigma_r}{(\sigma_r + a)} = -2\frac{\delta r}{r} \quad \text{.... (v)}$$

Integrating the equation (v)

$$\log_e(\sigma_r + a) = -2\log_e r + \log_e b, \text{ where } \log_e b \text{ is the integration constant}$$

$$= -\log_e r^2 + \log_e b = \log_e\left(\frac{b}{r^2}\right)$$

$$\text{ie., } \sigma_r + a = b/r^2$$

$$\therefore \sigma_r = \frac{b}{r^2} - a \qquad \text{.... (vi)}$$

Substituting the value of σ_r in equations (iv)

$$\sigma_\theta = \frac{b}{r^2} - a + 2a$$

$$\therefore \sigma_\theta = a + \frac{b}{r^2} \qquad \text{.... (vii)}$$

As radial stress is compressive stress, it is convenient to write the equation (vi) as

$$\sigma_r = a - \frac{b}{r^2} \qquad \text{.... (viii)}$$

Equation (viii) gives the radial stress at any radius r and equation (vii) gives the hoop stress (tangential stress) at any radius r. These two equations are called Lame's equations. The values of a and b are obtained from boundary conditions and are known as Lame's constant.

From Design Data Hand Book (Vol.I)

Hoop stress or Tangential stress in the cylinder wall at radius r,

$$\sigma_\theta = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} + \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2 (d_o^2 - d_i^2)} = a + \frac{b}{r^2} \qquad \text{.... 7.17 (DDHB)}$$

$$\sigma_\theta = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{4r^2}\right) \text{ when } p_o = o \qquad \text{.... 7.19 (DDHB)}$$

Maximum tangential stress at the inner surface $\sigma_{\theta_{Max}} = \frac{p_i (d_i^2 + d_o^2)}{(d_o^2 - d_i^2)}$ .... 7.21 (DDHB)

Radial stress at radius r, $\sigma_r = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} - \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2 (d_o^2 - d_i^2)} = a - \frac{b}{r^2}$ .... 7.18 (DDHB)

$$\sigma_r = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{4r^2}\right) \text{ when } p_o = o \qquad \text{.... 7.20 (DDHB)}$$

Maximum radial stress $\sigma_{r_{max}} = -p_i$ .... 7.22 (DDHB)

Maximum shear stress at the inner surface of the cylinder under internal pressure $\tau_{max} = \frac{p_i d_o^2}{d_o^2 - d_i^2}$

.... 7.23 (DDHB)

Wall thickness of the cylinder for brittle material $h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta + p_i}{\sigma_\theta - p_i}} - 1\right]$ .... 7.24 (DDHB)

Wall thickness of the cylinder for ductile material $h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta}{\sigma_\theta - 2p_i}} - 1\right]$ .... 7.25 (DDHB)

2.9 WALL THICKNESS OF CYLINDER

When the material of the cylinder is brittle, such as cast iron, cast steel etc, Lame's equation is used to determine the wall thickness. It is based on the maximum principal stress theory of failure. ie., Maximum principal stress is equated to the permissible stress of the material.

We know $\sigma_r = -p_i$ .... 7.22 (DDHB)

$$\sigma_0 = p_i \frac{(d_o^2 + d_i^2)}{(d_o^2 - d_i^2)} \quad \text{.... 7.21 (DDHB)}$$

$$\sigma_l = \frac{p_i d_i^2}{(do^2 - di^2)}$$

$\therefore \sigma_0 > \sigma_l > \sigma_r$

Hence σ_0 is the criterion of design

$$\text{ie., } \frac{\sigma_0}{p_i} = \frac{do^2 + di^2}{do^2 - di^2}$$

Substituting the values of principal stresses and by simplification

$$\text{we get, } \frac{d_o}{d_i} = \sqrt{\frac{\sigma_\theta + p_i}{\sigma_\theta - p_i}}$$

$\therefore$ Thickness of cylinder wall $h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta + p_i}{\sigma_\theta - p_i}} - 1\right]$ ($\because d_o = d_i + 2h$) .... 7.24 (DDHB)

When the material of the cylinder is ductile, such as mild steel or alloy steel, maximum principal strain theory or St. Venant's theory of failure is used as a criterion to indicate failure. According to this theory, failure will occur whenever a principal strain reaches a limiting value as determined from the standard tension test. The three principal stresses at the inner surface of the cylinder are as follows.

$$\sigma_r = -p_i$$

$$\sigma_0 = p_i\left(\frac{d_o^2+d_i^2}{d_o^2-d_i^2}\right) \text{ where do} = d_i + 2h$$

$$\sigma_l = \frac{p_i\, d_i^2}{\left(d_o^2-d_i^2\right)}$$

According to this theory for tri axial stress state the design equation is,

$$\frac{\sigma_{y_t}}{nE} = \frac{\sigma_\theta}{E} - \frac{\upsilon}{E}(\sigma_r + \sigma_l) \quad (\therefore\ \sigma_0 > \sigma_l > \sigma_r)$$

where n = FOS ; E = Young's modulus of the material ; υ = Poisson's ratio

ie., $\sigma_\theta' = \sigma_\theta - \upsilon(\sigma_r + \sigma_l)$ where σ_θ' = Permissible stress in tension = $\frac{\sigma_{y_t}}{n}$ or $\frac{\sigma_{u_t}}{n}$

Substituting the values of principal stresses and by simplification we get,

$$h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta' + (1-2\upsilon)p_i}{\sigma_\theta' - (1+\upsilon)p_i}} - 1\right] \quad \text{.... 7.28 (DDHB)}$$

Equation 7.28 (DDHB) is Clavarino's equation and it is applicable to cylinders with closed ends and made of ductile material.

When the cylinder ends are open, $\sigma_l = 0$

$\therefore\ \sigma_\theta' = \sigma_\theta - \upsilon\,\sigma_r$ where σ_θ' = Permissible stress in tension = $\frac{\sigma_{y_t}}{n}$ or $\frac{\sigma_{u_t}}{n}$

Substituting the values of principal stresses and by simplification we get,

$$h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta' + (1-\upsilon)p_i}{\sigma_\theta' - (1+\upsilon)p_i}} - 1\right] \quad \text{.... 7.31 (DDHB)}$$

Equation 7.31 (DDHB) is Birnie's equation and it is applicable to cylinders with open ends and made of ductile material.

According to Lame's theory for ductile material, maximum shear stress theory is used.

According to this theory, the maximum shear stress at any point in a strained body is equal to one half the algebraic difference of the maximum and minimum principal stresses at that point.

Maximum principal stress at the inner surface $\sigma_1 = \sigma_\theta = p_i\left(\frac{d_o^2+d_i^2}{d_o^2-d_i^2}\right)$ $(\therefore\ \sigma_\theta > \sigma_l > \sigma_r)$

Minimum principal stress at the inner surface $\sigma_2 = \sigma_r = -p_i$ $(\therefore\ \sigma_r < \sigma_l < \sigma_0)$

$\therefore$ Maximum shear stress $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$

Substituting the values of principal stresses and by simplification, we get

$$h = \frac{d_i}{2}\left[\sqrt{\frac{\tau_{max}}{\tau_{max} - p_i}} - 1\right]$$

$$\text{ie., } h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_0}{\sigma_\theta - 2p_i}} - 1\right] \left(\because \tau_{max} = \frac{\sigma_0}{2}\right) \qquad \text{.... 7.25 (DDHB)}$$

Example : 2.4

A cast iron cylinder of internal diameter 500 mm and 75 mm thick is filled with a fluid of pressure 6 N/mm². Determine the tangential and radial stresses at the inner, middle and outer surface. Also sketch the tangential stress and radial stress distribution across its thickness.

Data :

$$d_i = 500 \text{ mm}; \; d_o = 500 + 2 \times 75 = 650 \text{ mm}; \; p_i = 6 \text{ N/mm}^2 \; p_o = o$$

Solution :

Tangential stress in the cylinder wall at radius r

$$\sigma_\theta = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{4r^2}\right) \text{ when } p_o = o \qquad \text{.... 7.19 (DDHB)}$$

$$\text{When } r = r_i = \frac{500}{2} = 250 \text{ mm}$$

$$\sigma_{\theta_i} = \frac{6 \times 500^2}{\left(650^2 - 500^2\right)}\left(1 + \frac{650^2}{4 \times 250^2}\right) = 23.39 \text{ N/mm}^2$$

$$\text{When } r = r_m = \frac{250 + 325}{2} = 287.5 \text{ mm}$$

$$\sigma_{\theta_m} = \frac{6 \times 500^2}{\left(650^2 - 500^2\right)}\left(1 + \frac{650^2}{4 \times 287.5^2}\right) = 19.81 \text{ N/mm}^2$$

When $r = r_o = 325$ mm

$$\sigma_{\theta_o} = \frac{6 \times 500^2}{\left(650^2 - 500^2\right)}\left(1 + \frac{650^2}{4 \times 325^2}\right) = 17.39 \text{ N/mm}^2$$

Radial stress in the cylinder wall at radius r

$$\sigma_r = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{4r^2}\right) \text{ when } p = 0 \qquad \text{.... 7.20 (DDHB)}$$

When $r = r_i = 250$ mm

$$\sigma_{r_i} = \sigma_{r_{max}} = -p_i = -6 \text{ N/mm}^2 \qquad \text{.... 7.22 (DDHB)}$$

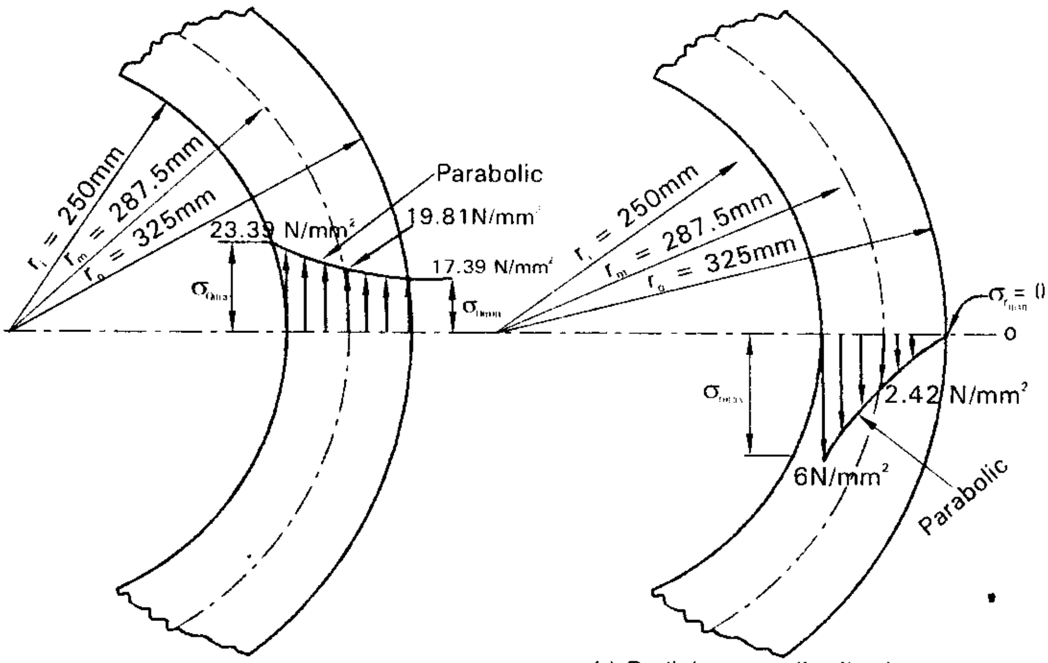
When $r = r_m = 287.5$ mm

$$\sigma_{r_m} = \frac{6 \times 500^2}{(650^2 - 500^2)}\left(1 - \frac{650^2}{4 \times 287.5^2}\right) = -2.42 \text{ N/mm}^2$$

When $r = r_o = 325$ mm

$$\sigma_{r_o} = 0$$

Tangential stress and radial stress distribution across the section is as shown in Fig. 2.5.



a) Tangential stress distribution b) Radial stress distribution

Fig. 2.5

Example : 2.5

A hydraulic press has a maximum capacity of 10 kN. The friction due to piston packings is equivalent to 10% of its capacity. The cylinder is made of cast iron whose ultimate tensile strength is 240 MPa. Diameter of the piston is 50 mm and factor of safety is 4. Determine the wall thickness of the cylinder.

Data :

σ_{u_t} = 240 MPa; D_i = 50 mm; FOS = 4

Solution :

$$\text{Total force on the piston } F = 10 + \frac{10}{100} \times 10 = 11 \text{ kN} = 11000 \text{ N}$$

$$\text{Also, } F = \frac{\pi}{4} D_i^2 \times p_i$$

$$\text{ie., } 11000 = \frac{\pi}{4} \times 50^2 \times p_i$$

∴ Pressure inside the cylinder $p_i = 5.6 \text{ N/mm}^2$

$$\text{Permissible tensile stress } \sigma_0 = \frac{\sigma_{ut}}{n} = \frac{240}{4} = 60 \text{ N/mm}^2$$

By Lame's equation for brittle materials,

$$\text{Wall thickness of cylinder } h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_0 + p_i}{\sigma_0 - p_i}} - 1\right] = \frac{50}{2}\left[\sqrt{\frac{60+5.6}{60-5.6}} - 1\right] = 5.147 \text{ mm} \quad \text{.... 17.4 (DDHB)}$$

take h = 6 mm

Example : 2.6

Determine the thickness of metal necessary for a cylinder of internal diameter 160 mm to withstand an internal fluid pressure of 8 N/mm². The maximum tangential stress in the section is not to exceed 35 N/mm². The material may be assumed as a brittle material.

Data :

d_i = 160 mm; p_i = 8 N/mm²; σ_θ = 35 N/mm²

Solution :

By Lame's equation for brittle materials,

$$\text{Wall thickness of cylinder } h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_0 + p_i}{\sigma_0 - p_i}} - 1\right] \quad \text{.... 17.24 (DDHB)}$$

$$= \frac{160}{2}\left[\sqrt{\frac{35+8}{35-8}} - 1\right] = 20.958 \text{ mm} \cong 21 \text{ mm}$$

Example : 2.7

The inside diameter of a cylindrical tank is 200 mm. The gas pressure inside the cylinder is 10 MPa. The tank is made of carbon steel whose ultimate tensile strength is 400 N/mm² and the factor of safety is 5. Find the wall thickness of the cylinder. Poisson's ratio of the material is 0.3.

Data :

$$d_i = 200 \text{ mm}; \; p_i = 10 \text{ Mpa}; \; \sigma_{ut} = 400 \text{ N/mm}^2; \text{ Fos} = 5 \; ; \upsilon = 0.3$$

Solution :

The cylinder is a closed cylinder since it stores gas under pressure.

$$\text{Permissible stress } \sigma_\theta' = \sigma_\theta = \frac{\sigma_{ut}}{FOS} = \frac{400}{5} = 80 \text{ N/mm}^2$$

By Lame's equation for ductile materials,

$$\text{Wall thickness of cylinder h} = \frac{d_i}{2}\left(\sqrt{\frac{\sigma_\theta}{\sigma_\theta - 2p_i}} - 1\right) \qquad \text{.... 7.25 (DDHB)}$$

$$= \frac{200}{2}\left[\sqrt{\frac{80}{(80 - 2\times 10)}} - 1\right] = 15.47 \text{ mm} \cong 16 \text{ mm}$$

By Clavarino's equation for ductile materials,

$$\text{Wall thickness of closed cylinder h} = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta' + (1-2\upsilon)p_i}{\sigma_\theta' - (1+\upsilon)p_i}} - 1\right] \qquad \text{.... 7.28 (DDHB)}$$

$$= \frac{200}{2}\left[\sqrt{\frac{80 + (1 - 2\times 0.3)10}{80 - (1+0.3)10}}\right] = 11.97 \text{ mm} \cong 12 \text{ mm}$$

Example : 2.8

A seamless steel pipe of 150 mm internal diameter is subjected to internal pressure of 10 MPa. The pipe is made of steel whose tensile strength at the yield point is 240 N/mm² and the factor of safety is 3. Poisson's ratio of the material is 0.27 Determine the wall thickness of the pipe.

Data :

$$d_i = 150 \text{ mm}; \; p_i = 10 \text{ MPa}; \; \sigma_{yt} = 240 \text{ N/mm}^2; \text{ FOS} = 3; \upsilon = 0.27$$

Solution :

It is an open cylinder as it is a pipe.

$$\text{Permissible stress } \sigma_\theta' = \sigma_\theta = \frac{\sigma_{yt}}{Fos} = \frac{240}{3} = 80 \text{ N/mm}^2$$

By Lame's equation for ductile materials, Wall thickness of cylinder $h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta}{\sigma_\theta - 2p_i}} - 1\right]$ ----- 7.25 (DDHB)

$$= \frac{150}{2}\left[\sqrt{\frac{80}{80 - 2\times 10}} - 1\right] = 11.6 \text{ mm} \cong 12 \text{ mm}$$

By Birnie's equation for ductile materials,

Wall thickness of an open cylinder $h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta' + (1-\upsilon)p_i}{\sigma_\theta' - (1+\upsilon)p_i}} - 1\right]$ ----- 7.31 (DDHB)

$$= \frac{150}{2}\left[\sqrt{\frac{80 + (1-0.27)10}{80 - (1+0.27)10}} - 1\right] = 10.42 \text{ mm} \cong 11\text{mm}.$$

Example : 2.9

A cast iron cylindrical pipe of outside diameter 300 mm and inside diameter 200 mm is subjected to an internal fluid pressure of 20 N/mm² and external fluid pressure of 5 N/mm². Determine the tangential and radial stresses at the inner, middle and outer surface. Also sketch the tangential stress and radial stress distribution across its thickness

Data :

$d_o = 300$ mm; $d_i = 200$ mm; $p_i = 20$ N/mm²; $p_o = 5$ N/mm²

Solution :

Lame's general expression for tangential stress in the cylinder wall at radius r,

$$\sigma_\theta = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} + \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2\left(d_o^2 - d_i^2\right)}$$ ----- 7.17 (DDHB)

When $r = r_i = \frac{200}{2} = 100$ mm

$$\sigma_{\theta_i} = \frac{20\times 200^2 - 5\times 300^2}{300^2 - 200^2} + \frac{200^2 \times 300^2 (20-5)}{4\times 100^2\left(300^2 - 200^2\right)} = 34 \text{ N/mm}^2$$

When $r = r_m = \frac{100+150}{2} = 125$ mm

$$\sigma_{\theta_m} = \frac{20\times 200^2 - 5\times 300^2}{300^2 - 200^2} + \frac{200^2 \times 300^2 (20-5)}{4\times 125^2\left(300^2 - 200^2\right)} = 24.28 \text{ N/mm}^2$$

When $r = r_o = \frac{300}{2} = 150$ mm

$$\sigma_{\theta_o} = \frac{20\times200^2-5\times300^2}{300^2-200^2}+\frac{200^2\times300^2(20-5)}{4\times150^2\left(300^2-200^2\right)} = 19\ \text{N/mm}^2$$

Lame's general expression for radial stress in the cylinder wall at radius r,

$$\sigma_r = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2-d_i^2}-\frac{d_i^2 d_o^2(p_i-p_o)}{4r^2\left(d_o^2-d_i^2\right)} \qquad \text{----- 7.18 (DDHB)}$$

When $r = r_i = 100$ mm

$$\sigma_{r_i} = \sigma_{r_{max}} = -p_i = -20\ \text{N/mm}^2$$

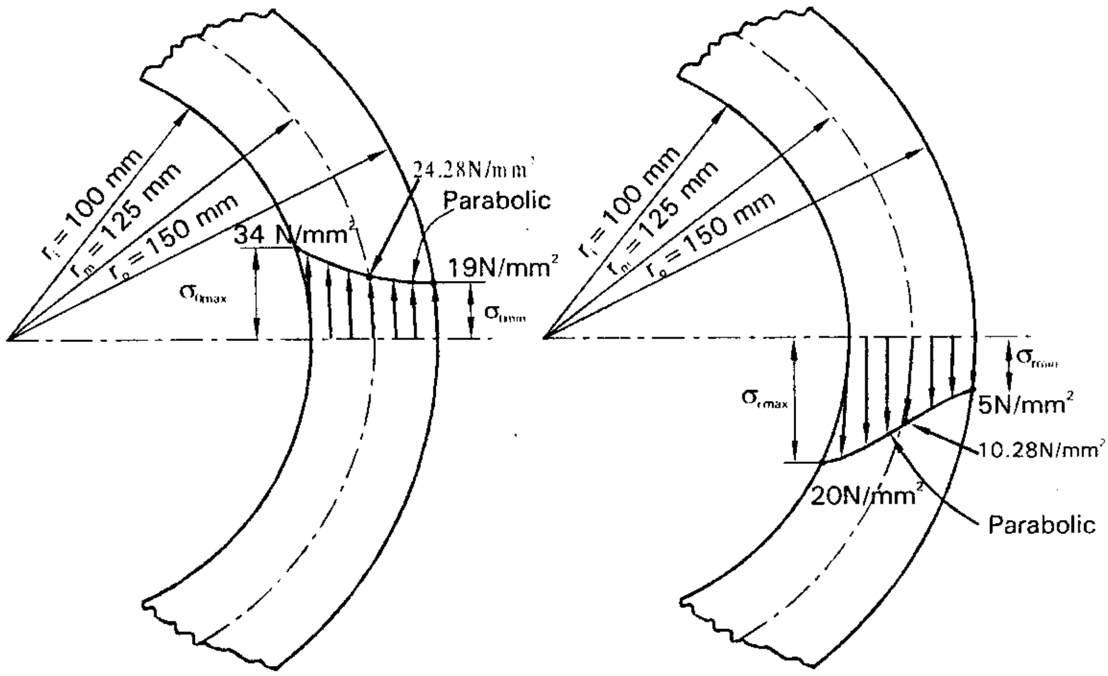
When $r = r_o = 150$ mm

$$\sigma_{r_o} = \sigma_{r_{min}} = -p_o = -5\ \text{N/mm}^2$$

When $r = r_m = 125$ mm

$$\sigma_{r_m} = \frac{20\times200^2-5\times300^2}{300^2-200^2}-\frac{200^2\times300^2(20-5)}{4\times125^2\left(300^2-200^2\right)} = -10.28\ \text{N/mm}^2$$

Tangential stress and radial stress distribution across the section is as shown in Fig 2.6



a) Tangential stress distribution

b) Radial stress distribution

Fig 2.6

2.10 COMPOUND CYLINDER

According to Lame's equation, the thickness of a cylindrical shell is given by

$$h = \frac{d_i}{2}\left(\sqrt{\frac{\sigma_\theta + p_i}{\sigma_\theta - p_i}} - 1\right) \qquad \text{----- 7.24 (DDHB)}$$

If the internal pressure p_i is equal or greater than the allowable tensile stress σ_θ, then the root value is imaginary. Therefore it is not possible to design a cylinder if the internal pressure is greater than the allowable stress. This ill effect will be overcome by prestressing the cylinder before using it in service. This may be done by the following methods.

(i) By using compound cylinder

In a compound cylinder, the outer cylinder is shrunk fit over the inner cylinder by heating and cooling. On cooling a contact pressure is developed at the junction of the two cylinders which induces compressive tangential stress in the inner cylinder and tensile tangential stress in the outer cylinder. When the cylinder is loaded, the compressive stresses are first relieved and than tensile stresses are induced. Thus a compound cylinder is more effective in resisting higher internal pressure than a single cylinder with the same overall dimensions.

(ii) By overloading the cylinder before it is put into service i.e., By using the theory of plasticity

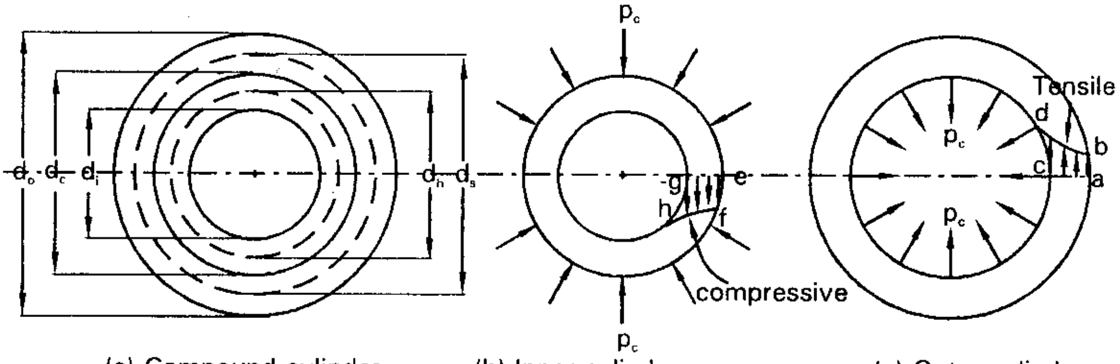
In this method, a temporary high internal pressure is applied till the plastic stage is reached near the inside of the cylinder wall while the outer portion is still in the elastic range. When the pressure is released, the outer portion contracts exerting pressure on the inner portion which has undergone permanent deformation. This induces residual compressive stresses at the inner surface and tensile stresses at the outer surface.

(iii) A wire under tension is closely wound around the cylinder, which results in residual compressive stress.

This process of pre-stressing the cylinder before using it in service is called "autofrettage". Autofrettage not only increases the pressure capacity of the cylinder but also improves the endurance strength.

2.11 STRESSES IN COMPOUND CYLINDER

Fig 2.7 (a) shows a compound cylinder assembled with a shrink fit.



(a) Compound cylinder (b) Inner cylinder (c) Outer cylinder

Fig 2.7

Let d_i = Inside diameter of inner cylinder
d_o = Outside diameter of outer cylinder
d_c = Common diameter after shrink fit
d_s = Outside diameter of inner cylinder = Diameter of shaft
d_h = Inner diameter of outer cylinder = Diameter of hub
p_c = Contact pressure
σ_θ = Tangential stress
σ_r = Radial stress
Δd_i = Change in diameter of inner member
Δd_o = Change in diameter of outer member
δ = Total interference
μ = Coefficient of friction
υ = Poisson's Ratio = 0.3 for steel

When the outer cylinder is shrunk fit over the inner cylinder, a contact pressure p_c is developed at the junction of the two cylinders, (i.e., at radius r_c) The stresses due to this pressure may be determined by using Lame's equation.

(1) The general expression for tangential stress at radius r,

$$\sigma_\theta = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} + \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2 (d_o^2 - d_i^2)} \qquad \text{----- 7.17 (DDHB)}$$

Considering external pressure only

$$\sigma_\theta = -\frac{p_o d_o^2}{d_o^2 - d_i^2}\left(1 + \frac{d_i^2}{4r^2}\right) \qquad \text{----- (i)}$$

Considering the internal pressure only

$$\sigma_\theta = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{4r^2}\right) \qquad \text{----- (ii)}$$

(i) Tangential stress at the outside diameter of outer cylinder $\sigma_{\theta_{-(o)}}$

From equation (ii) substituting $p_i = p_c$, $r = \frac{d_o}{2}$ and $d_i = d_c$

$$\sigma_{\theta_{-(o)}} = \frac{p_c d_c^2}{d_o^2 - d_c^2}\left(1 + \frac{d_o^2}{d_o^2}\right) = \frac{2 p_c d_c^2}{d_o^2 - d_c^2} \qquad \text{----- 11.12(DDHB)}$$

It is tensile stress and is shown by the line ab in Fig 2.7 (c)

(ii) Tangential stress at the inside diameter of outer cylinder $\sigma_{\theta_{-oi}}$

From equation (ii) substituting $p_i = p_c$, $r = \frac{d_c}{2}$ and $d_i = d_c$

$$\sigma_{\theta_{-oi}} = \frac{p_c d_c^2}{d_o^2 - d_c^2}\left(1 + \frac{d_o^2}{d_c^2}\right) = p_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right) \quad \text{----- 11.13 (DDHB)}$$

It is tensile stress and is shown by the line cd in Fig 2.7 (c)

(iii) Tangential stress at the outside diameter of inner cylinder $\sigma_{\theta_{-io}}$

From equation (i) substituting $p_o = p_c$, $d_o = d_c$ and $r = \frac{d_c}{2}$

$$\sigma_{\theta_{-io}} = \frac{-p_c d_c^2}{d_c^2 - d_i^2}\left(1 + \frac{d_i^2}{d_c^2}\right) = -p_c\left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2}\right) \quad \text{----- 11.14 (DDHB)}$$

It is compressive stress and is shown by the line ef in Fig 2.7 (b)

(iv) Tangential stress at the inner diameter of inner cylinder $\sigma_{\theta_{-ii}}$

From equation (i), substituting $p_o = p_c$, $d_o = d_c$ and $r = \frac{d_i}{2}$

$$\sigma_{\theta_{-ii}} = -\frac{p_c d_c^2}{d_c^2 - d_i^2}\left(1 + \frac{d_i^2}{d_i^2}\right) = -\frac{2p_c d_c^2}{d_c^2 - d_i^2} \quad \text{----- 11.15 (DDHB)}$$

It is compressive stress and is shown by the line gh in Fig 2.7 (b)

(2) The General expression for radial stress at radius r,

$$\sigma_r = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} - \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2 (d_o^2 - d_i^2)} \quad \text{----- 7.18 (DDHB)}$$

Considering external pressure only,

$$\sigma_r = -\frac{p_o d_o^2}{d_o^2 - d_i^2}\left(1 - \frac{d_i^2}{4r^2}\right) \quad \text{----- (iii)}$$

Considering internal pressure only

$$\sigma_r = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{4r^2}\right) \quad \text{----- (iv)}$$

(i) Radial stress at the outside diameter of outer cylinder $\sigma_{r_{-oo}}$

From equation (iv), substituting $p_i = p_c$, $r = \frac{d_o}{2}$ and $d_i = d_c$,

$$\sigma_{r_{-oo}} = \frac{p_c d_c^2}{d_o^2 - d_c^2}\left(1 - \frac{d_o^2}{d_o^2}\right) = 0 \qquad \text{----- 11.16 (DDHB)}$$

(ii) Radial stress at the inside diameter of outer cylinder $\sigma_{r_{-oi}}$

From equation (iv) substituting $p_i = p_c$, $r = \frac{d_c}{2}$ and $d_i = d_c$

$$\sigma_{r_{-oi}} = \frac{p_c d_c^2}{d_o^2 - d_c^2}\left(1 - \frac{d_o^2}{d_c^2}\right) = -p_c \qquad \text{----- 11.17 (DDHB)}$$

(iii) Radial stress at the outside diameter of inner cylinder $\sigma_{r_{-io}}$

From equation (iii), substituting $p_o = p_c$, $d_o = d_c$ and $r = \frac{d_c}{2}$

$$\sigma_{r_{-io}} = \frac{-p_c d_c^2}{d_c^2 - d_i^2}\left(1 - \frac{d_i^2}{d_c^2}\right) = -p_c \qquad \text{----- 11.18 (DDHB)}$$

(iv) Radial stress at the inside diameter of inner cylinder $\sigma_{r_{-ii}}$

From equation (iii), substituting $p_o = p_c$, $d_o = d_c$ and $r = \frac{d_i}{2}$

$$\sigma_{r_{-ii}} = -\frac{p_c d_c^2}{d_c^2 - d_i^2}\left(1 - \frac{d_i^2}{d_i^2}\right) = 0 \qquad \text{----- 11.19 (DDHB)}$$

Radial stress distribution due to shrinkage fitting is shown in Fig 2.8

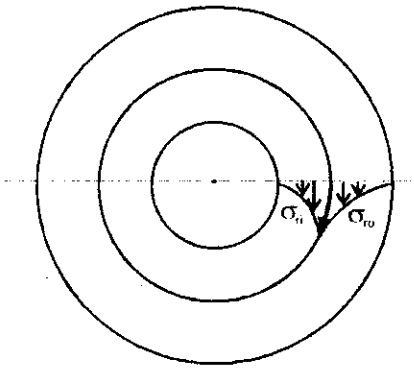


Fig 2.8 [Fig 11.2(b) DDHB]

(3) Change in dimensions

(i) Change in diameter of inner member $\Delta d_i = -\frac{p_c d_c}{E}\left[\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2} - \upsilon\right]$ ----- 11.2 (DDHB)

(ii) Change in diameter of outer member $\Delta d_o = \frac{p_c d_c}{E}\left[\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + \upsilon\right]$ ----- 11.3 (DDHB)

(iii) Total interference $\delta = \Delta d_o + \Delta d_i$

(iv) Diameter of shaft $d_s = d_c + \Delta d_i$

(v) Diameter of hub $d_h = d_c - \Delta d_o$

(4) **Axial force necessary to press the shaft into the hub $F_a = \pi\, d_c l\, \mu p_c$** ----- 11.22 (DDHB)

(5) Torque capacity of shrink fit $M_t = \frac{p d_c^2 l \mu p_c}{2} = F_a . \frac{d_c}{2}$ ----- 11.26 (DDHB)

(6) If the compound cylinder is subjected to an internal fluid pressure p_i and external pressure $p_o = 0$, then the general expression for tangential stress according to Lame's equation is,

$$\sigma_\theta = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{4r^2}\right) \quad \text{----- (v) [7.19 (DDHB)]}$$

(i) Tangential stress due to internal fluid pressure p_i at the outer surface of outer cylinder $\sigma'_{\theta_{oo}}$

From equation (v), substituting $r = \frac{d_o}{2}$

$$\sigma'_{\theta_{oo}} = \frac{p_i d_i^2}{d_o^2 - d_i^2} \cdot \left(1 + \frac{d_o^2}{d_o^2}\right) = \frac{2 p_i d_i^2}{d_o^2 - d_i^2}$$

It is tensile stress and is shown by the line ab' in Fig 2.9 (a)

(ii) Tangential stress due to internal fluid pressure p_i at the outer surface of inner cylinder or inner surface of outer cylinder $\sigma'_{\theta_{oi}}$ or $\sigma'_{\theta_{io}}$

From equation (vi, substituting $r = d_c/2$

$$\sigma'_{\theta_{oi}} = \sigma'_{\theta_{io}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{d_c^2}\right) = \frac{p_i d_i^2}{d_c^2}\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_i^2}\right)$$

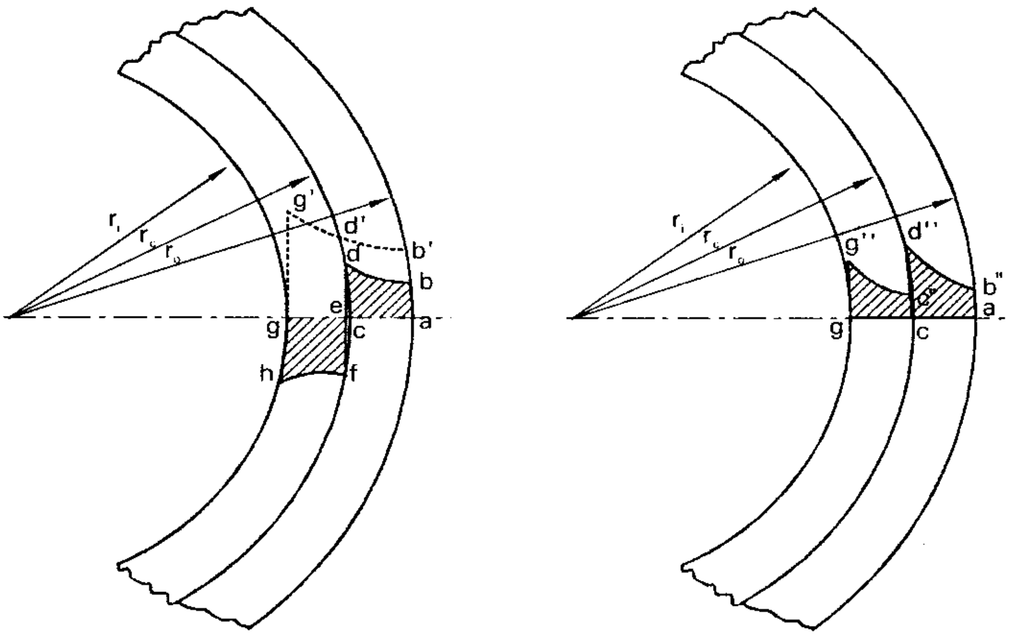
It is tensile stress and is shown by the line cd' in Fig. 2.9 (a)

(iii) Tangential stress due to internal fluid pressure p_i at the inner surface of inner cylinder $\sigma'_{\theta_{ii}}$

From equation (v), substituting $r = \frac{d_i}{2}$

$$\sigma'_{\theta_{ii}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{d_i^2}\right) = p_i\left(\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}\right)$$

It is tensile stress and is shown by the line gg' in Fig 2.9 (a)



a) Tangential stress distribution due to Shrinkage fitting and internal fluid pressure

a) Resultant tangential stress distribution due to Shrinkage fitting and internal fluid pressure

Fig 2.9

Resultant tangential stress at the outer surface of outer cylinder = $\sigma_{\theta_{-oo}} + \sigma'_{\theta_{oo}}$ = ab + ab_1

It is tensile stress and is shown by the line ab" in Fig 2.9 (b)

Resultant tangential stress at the inner surface of outer cylinder = $\sigma_{\theta_{-oi}} + \sigma'_{\theta_{oi}}$ = cd + cd'

It is tensile stress and is shown by the line cd" in Fig 2.9 (b).

Resultant tangential stress at the outer surface of inner cylinder = $\sigma_{\theta_{-io}} + \sigma'_{\theta_{io}}$ = ed' – ef

It is tensile stress and is shown by the line cc" in Fig 2.9 (b)

Resultant tangential stress at the inner surface of inner cylinder = $\sigma_{\theta_{-ii}} + \sigma'_{\theta_{ii}}$ = gg' – gh

It is tensile stress and is shown by the line gg" in Fig 2.9 (b)

(7) If the compound cylinder is subjected to an internal fluid pressure p_i and external pressure $p_o = 0$, then the general expression for radial stress according to Lame's equation is,

$$\sigma_r = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{4r^2}\right) \quad \text{----- (vi) [7.20 (DDHB)]}$$

(i) Radial stress due to internal fluid pressure p_i at the outersurface of outer cylinder $\sigma'_{r_{oo}}$

From equation (vi), substituting $r = \frac{d_o}{2}$

$$\sigma'_{r_{oo}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{d_o^2}\right) = 0$$

(ii) Radial stress due to internal fluid pressure p_i at the inner surface of outer cylinder or outer surface of inner cylinder $\sigma'_{r_{oi}}$ or $\sigma'_{r_{io}}$

From equation (vi), substituting $r = \frac{d_c}{2}$

$$\sigma'_{r_{oi}} = \sigma'_{r_{io}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{d_c^2}\right) = -\frac{p_i d_i^2}{d_c^2}\left(\frac{d_o^2 - d_c^2}{d_o^2 - d_i^2}\right)$$

It is compressive stress

(iii) Radial stress due to internal fluid pressure p_i at the innersurface of inner cylinder $\sigma'_{r_{ii}}$

From equation (vi), substituting $r = \frac{d_i}{2}$

$$\sigma'_{r_{ii}} = \frac{p_i d_i^2}{(d_o^2 - d_i^2)}\left(1 - \frac{d_o^2}{d_i^2}\right) = -p_i$$

It is compressive stress.

Resultant radial stress at the inner surface of inner cylinder = $\sigma_{r_{-ii}} + \sigma'_{r_{ii}}$

It is compressive stress

Resultant radial stress at the outer surface of the outer cylinder = $\sigma_{r_{-oo}} + \sigma'_{r_{oo}}$

Resultant radial stress at the inner surface of the outer cylinder or Resultant radial stress at the outer surface of inner cylinder = $\sigma_{r_{-oi}} + \sigma'_{r_{oi}}$.

It is compressive stress.

Resultant radial stress distribution due to shrinkage fitting and internal fluid pressure is shown in Fig 2.10.

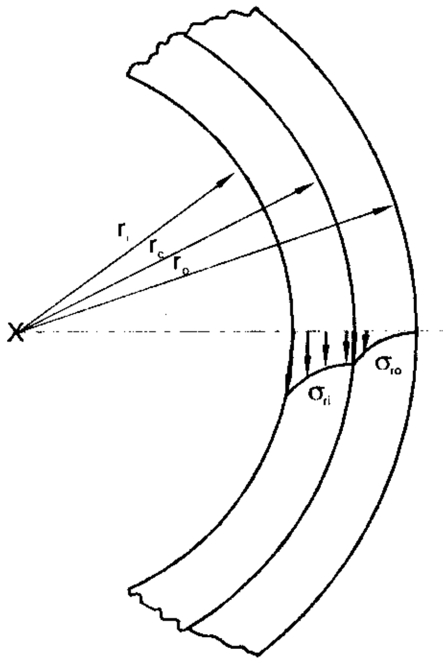


Fig 2.10

Example : 2.10

A carbon steel C50 barrel with diameter 25 mm and 50 mm is to be shrink fitted into another barrel with diameter 50 mm and 75 mm. The tangential stress developed at the inner fibre of the outer barrel due to shrink fitting may be limited to 70 N/mm². Determine,

(i) Contact pressure

(ii) Original diameter at contact before shrink fitting

(iii) Resulting stress distribution due to shrink fitting. E = 21 × 10⁴ N/mm²; υ = 0.28

Data :

$d_i = 25$ mm; $d_c = 50$ mm; $d_o = 75$ mm; $\upsilon = 0.28$;
$E = 21 \times 10^4$ N/mm²; $\sigma_{\theta-oi} = 70$ N/mm².

Solution :

(i) Contact pressure

Tangential stress at the inside diameter of outer cylinder

$$\sigma_{\theta-oi} = p_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right) \quad \text{----- 11.13 (DDHB)}$$

$$\text{i.e.,}\ 70 = \frac{p_c(75^2 + 50^2)}{(75^2 - 50^2)}$$

$\therefore$ Contact pressure $p_c = 26.92$ N/mm²

(ii) Original diameter at contact before shrink fitting

Change in diameter of inner member $\Delta d_i = \dfrac{-p_c d_c}{E}\left[\dfrac{d_c^2 + d_i^2}{d_c^2 - d_i^2} - \upsilon\right]$

$$= \frac{-26.92 \times 50}{21 \times 10^4}\left[\frac{50^2 + 25^2}{50^2 - 25^2} - 0.28\right] = -8.888 \times 10^{-3}\ \text{mm} \cong -0.009\ \text{mm}$$

$\therefore$ Original outside diameter of inner member $d_s = d_c + \Delta d_i = 50 + 0.009 = 50.009$ mm

Change in diameter of outer member $\Delta d_o = \dfrac{p_c d_c}{E}\left[\dfrac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + \upsilon\right]$ .... 11.3 (DDHB)

$$= \frac{26.92 \times 50}{21 \times 10^4}\left[\frac{75^2 + 50^2}{75^2 - 50^2} + 0.28\right] = 0.018\ \text{mm}$$

$\therefore$ Original inner diameter of outer member $d_h = d_c - \Delta d_o = 50 - 0.018 = 49.982$ mm

(iii) (a) **Tangent stresses due to shrink fit**

Tangential stress at outside diameter of outer member $\sigma_{\theta-oo} = \dfrac{2p_c d_c^2}{d_o^2 - d_c^2}$ .... 11.12 (DDHB)

$$= \frac{2 \times 26.92 \times 50^2}{75^2 - 50^2} = 43.072\ \text{N/mm}^2$$

Tangential stress at inside diameter of outer member $\sigma_{\theta-oi} = 70$ N/mm² (Given)

Tangential stress at outside diameter of inner member $\sigma_{\theta-io} = -\dfrac{p_c(d_c^2 + d_i^2)}{(d_c^2 - d_i^2)}$ .... 11.14 (DDHB)

$$= \frac{-26.92\left(50^2+25^2\right)}{\left(50^2-25^2\right)} = -44.8 \text{ N/mm}^2$$

Tangential stress at inside diameter of inner member $\sigma_{\theta-ii} = -\dfrac{2p_c d_c^2}{\left(d_c^2-d_i^2\right)}$ .... 11.15 (DDHB)

$$= -\frac{2\times 26.92\times 50^2}{\left(50^2\times 25^2\right)} = -71.78 \text{ N/mm}^2$$

(b) Radial stresses due to shrink fit

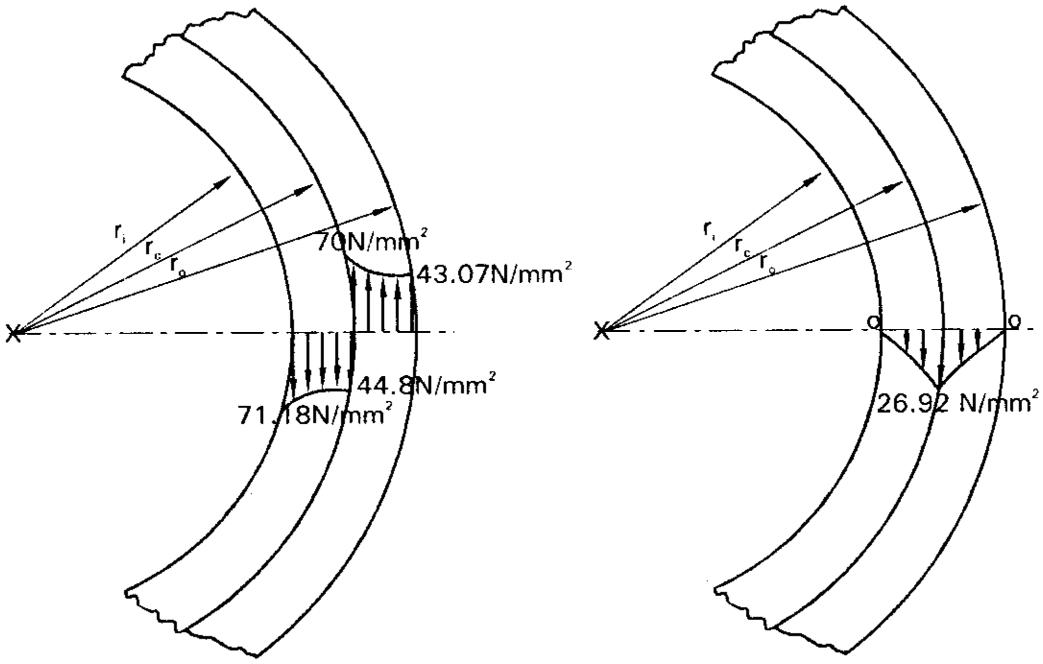
Redial stress at outside diameter of outer member $\sigma_{r-oo} = 0$ .... 11.16 (DDHB)

Radial stress at inside diameter of outer member $\sigma_{r-oi} = -p_c = -26.92$ N/mm² .... 11.17 (DDHB)

Radial stress at outside diameter of inner member $\sigma_{r-io} = -p_c = -26.92$ N/mm² .... 11.18 (DDHB)

Radial stress at inside diameter of inner member $\sigma_{r-ii} = 0$ .... 11.19 (DDHB)

Tangential stress distribution and radial stress distribution due to shrinkage fitting are shown in Fig. 2.11 (a) and 2.11 (b) respectively.



(a) Tangential stress distribution due to shrink fitting

(b) Radial stress distribution due to shrink fitting

Fig. 2.11

Example : 2.11

A solid shaft of 125 mm diameter is to be pressed into a steel flange which has an outside diameter of 150 mm and length of 100 mm. E = 21 × 10⁴ N/mm², υ = 0.3. Determine,

(i) Proper size of bore so that the maximum stress in the bore does not exceed 160N/mm²

(ii) Pressure between hub and shaft.

(iii) Force required to press the parts together.

(iv) Torque capacity of the press fit.

Data :

d_c =125 mm; d_o = 150 mm; d_i = 0 (∵ Solid shaft) ; $\sigma_{\theta-oi}$ = 160 N/mm²; E = 21 × 10⁴ N/mm²; υ = 0.3

Solution :

(i) Pressure between hub and shaft

Tangential stress at the inside diameter of outer member $\sigma_{\theta-oi} = p_c\left(\frac{d_o^2+d_c^2}{d_o^2-d_c^2}\right)$ .... 11.13 (DDHB)

$$\text{ie., } 160 = p_c\left(\frac{150^2+125^2}{150^2-125^2}\right)$$

∴ Pressure between hub and shaft p_c = 28.85 N/mm²

(ii) Size of bore

Change in diameter of outer member $\Delta d_o = \frac{p_c d_c}{E}\left(\frac{d_o^2+dc^2}{d_o^2-d_c^2}+\upsilon\right)$ .... 11.3 (DDHB)

$$= \frac{28.85\times125}{21\times10^4}\left(\frac{150^2+125^2}{150^2-125^2}+0.3\right) = 0.1 \text{ mm}$$

∴ Proper size of bore $d_h = d_c - \Delta d_o$ = 125 − 0.1 = 124.9 mm

(iii) Force required to press the parts together

Axial force necessary to press shaft into hub $F_a = \pi d_c l \mu p_c$ .... 11.22 (DDHB)

$$= \pi \times 125 \times 100 \times 0.1 \times 28.85 = 113293.7\text{N} \cong 113.3 \text{ kN}$$

(∵ For press fit μ = 0.1)

(iv) Torque Capacity

Torque of press fit $M_t = \frac{\pi d_c^2 l \mu p_c}{2} = F_a.\frac{d_c}{2} = 113.3 \times \frac{125}{2} = 7081.25 \text{ kNmm}$ .... 11.26 (DDHB)

$$= 7.08125 \text{ kNm} \cong 7.1 \text{ kNm}.$$

Example : 2.12

Design a shrink fit joint to join two cylinders of diameter 150 mm × 200 mm and 200 mm × 250 mm. Maximum tangential stress in the components due to shrink fitting is to be limited to 40 Mpa. Also determine the axial force necessary to dis-engage the joint if the length of the joint is 200 mm and the maximum power that can be transmitted at a rated speed of 1000 rpm. The material of the cylinder has a modulus of elasticity 210 GPa and Poisson's ratio 0.3 [Bangalore University, Feb'96, Aug'92]

Data :

d_i =150 mm; d_c = 200 mm; d_o = 250 mm; $\sigma_{\theta-oi}$ = 40 MPa = 40 N/mm^2 ; l = 200 mm ; n = 1000 rpm; E = 210 GPa = 210 × 10^3 N/mm^2 ; υ = 0.3

Solution :

(i) Contact Pressure

Tangential stress at the inside diameter of outer cylinder

$$\sigma_{\theta-oi} = p_c\left(\frac{d_o^2+d_c^2}{d_o^2-d_c^2}\right) \qquad \text{.... 11.13 (DDHB)}$$

$$\text{ie., } 40 = p_c\left(\frac{250^2+200^2}{250^2-200^2}\right)$$

$\therefore$ Pressure at the contact point or junction p_c = 8.78 N/mm^2

(ii) Tangential stress due to shrink fit

Tangential stress at the outside diameter of outer cylinder $\sigma_{\theta-oo} = \dfrac{2p_c d_c^2}{d_o^2-d_c^2}$ .... 11.12 (DDHB)

$$= \frac{2\times 8.78\times 200^2}{250^2-200^2} = 31.22 \text{ N/mm}^2$$

Tangential stress at the inside diameter of outer cylinder $\sigma_{\theta-oi}$ = 40 N/mm^2 (given)

Tangential stress at the outside diameter of inner cylinder $\sigma_{\theta-io} = -p_c\left(\dfrac{d_c^2+d_i^2}{d_c^2-d_i^2}\right)$ .... 11.14 (DDHB)

$$= -8.78\left(\frac{200^2+150^2}{200^2-150^2}\right) = -31.36 \text{ N/mm}^2$$

Tangential stress at the inside diameter of inner cylinder $\sigma_{\theta-ii} = -\dfrac{2p_c d_c^2}{d_c^2-d_i^2}$ .... 11.15 (DDHB)

$$= -\frac{2\times 8.78\times 200^2}{\left(200^2-150^2\right)} = -40.137 \text{ N/mm}^2$$

Tangential stress distribution due to shrinkage fitting is shown in Fig. 2.12 (a)

(iii) Radial stress due to shrink fit

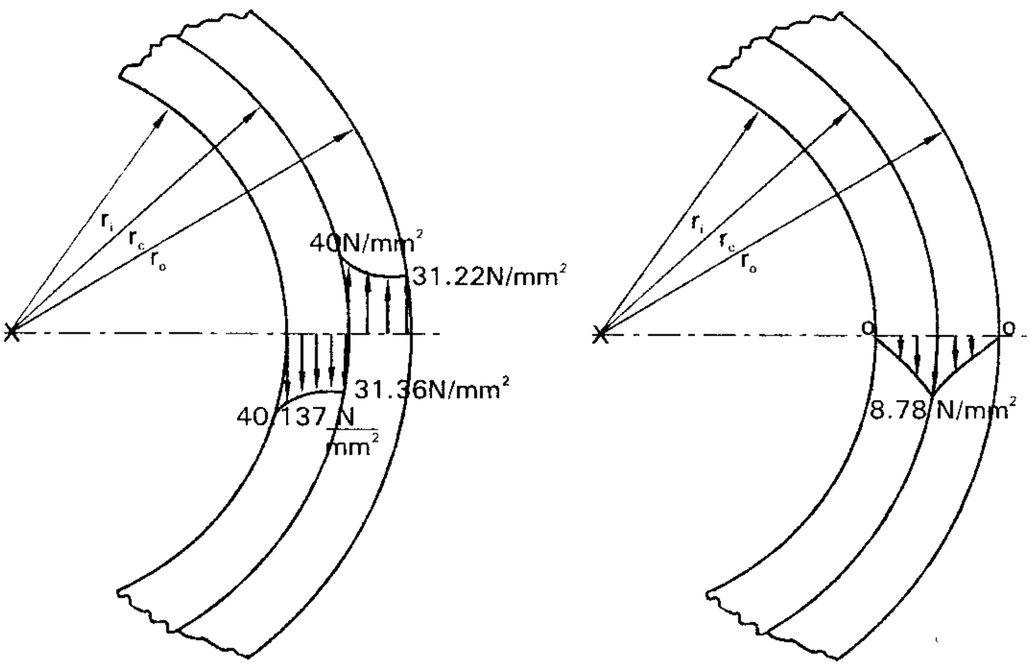
Radial stress at the outside diameter of outer cylinder $\sigma_{r-oo} = 0$ .... 11.16 (DDHB)

Radial stress at the inside diameter of outer cylinder $\sigma_{r-oi} = -p_c = -8.78$ N/mm² .... 11.17 (DDHB)

Radial stress at the outside diameter of inner cylinder $\sigma_{r-io} = -p_c = -8.78$ N/mm² .... 11.18 (DDHB)

Radial stress at the inside diameter of inner cylinder $\sigma_{r-ii} = 0$ .... 11.19 (DDHB)

Radial stress distribution due to shrinkage fitting is shown in Fig. 2.12 (b)



(a) Tangential stress distribution due to shrink fitting

(b) Radial stress distribution due to shrink fitting

Fig. 2.12

(iv) Change in dimensions

Change in diameter of inner member $\Delta di = -\frac{p_c d_c}{E}\left[\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2} - \upsilon\right]$ .... 11.2 (DDHB)

$$= -\frac{8.78 \times 200}{210 \times 10^3}\left[\frac{200^2 + 150^2}{200^2 - 150^2} - 0.3\right] = -0.027 \text{ mm} \cong -0.03 \text{ mm}$$

Change in diameter of outer member $\Delta d_o = \frac{p_c d_c}{E}\left[\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + \upsilon\right]$ .... 11.3 (DDHB)

$$= \frac{8.78 \times 200}{210 \times 10^3}\left[\frac{250^2 + 200^2}{250^2 - 200^2} + 0.3\right] = 0.04 \text{ mm}$$

$\therefore$ Total interference $\delta = \Delta d_o + \Delta d_i = 0.03 + 0.04 = 0.07$ mm .... 11.4 (DDHB)

Original inner diameter of outer member $d_h = d_c - \Delta d_o = 200 - 0.04 = 199.96$ mm

Original outer diameter of inner member $d_s = d_c + \Delta d_i = 200 + 0.03 = 200.03$ mm

(v) Force necessary to dis-engage the joint

Axial force necessary to dis-engage the joint $F_a = \pi d_c l \mu p_c$ .... 11.22 (DDHB)

$= \pi \times 200 \times 200 \times 0.125 \times 8.78$ [For shrink fit $\mu = 0.125$]

$= 137915.9$ N

(vi) Torque Capacity

Torque of shrink fit $M_t = \frac{\pi d_c^2 l \mu p_c}{2} = F_a \cdot \frac{d_c}{2}$ .... 11.25 (DDHB)

$$= 137915.9 \times \frac{200}{2} = 13791590 \text{ Nmm} \cong 13.8 \text{ kNm}$$

(vii) Power transmitted

Torque of the shrink fit $M_t = 9550 \times 1000 \times \frac{N}{n}$ where M_t in Nmm

ie., $13791590 = 9550 \times 1000 \times \frac{N}{1000}$

$\therefore$ Power transmitted $N = 1444.15$ kW

Example : 2.13

A 440 mm outer diameter, 250 mm inner diameter and 300 mm long steel hub is to be shrink on to a 250 mm diameter steel shaft. If the torque to be transmitted is 300 kNm and $\mu = 0.18$, determine the amount of interference required. [Bangalore University, Sep'98]

Data :

$d_o = 440$ mm; $d_c = 250$ mm; $d_i = 0$ ($\therefore$ solid shaft); $l = 300$ mm; $M_t = 300$ kNm $= 300 \times 10^6$ Nmm ; $\mu = 0.18$

Solution :

(i) Contact pressure

Torque of shrink fit $M_t = \frac{\pi d_c^2 l \mu p_c}{2}$

$$\text{ie., } 300 \times 10^6 = \frac{\pi \times 250^2 \times 300 \times 0.18 \times p_c}{2}$$

$$\therefore \text{ Contact pressure } p_c = 56.6 \text{ N/mm}^2$$

(ii) Total interference

$$\text{Change in diameter of inner member } \Delta d_i = -\frac{p_c d_c}{E}\left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2} - \upsilon\right) = -\frac{p_c d_c}{E}[1 - \upsilon] \; (\because d_i = o)$$

... 11.2 (DDHB)

From Table 2.10 (New DDHB) for steel

Modulus of elasticity $E = 206 \text{ GPa} = 206 \times 10^3 \text{ N/mm}^2$

Poisson's ratio $\upsilon = 0.292$

$$\therefore \Delta d_i = -\frac{56.6 \times 250}{206 \times 10^3}[1 - 0.292] = 0.0486 \text{ mm}$$

$$\text{Change in diameter of outer member } \Delta d_o = \frac{p_c d_c}{E}\left[\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + \upsilon\right] \quad \text{.... 11.3 (DDHB)}$$

$$= \frac{56.6 \times 250}{206 \times 10^3}\left[\frac{440^2 + 250^2}{440^2 - 250^2} + 0.292\right] = 0.1542 \text{ mm}$$

$$\therefore \text{ Total interference } \delta = \Delta d_i + \Delta d_o = 0.0486 + 0.1542 = 0.2028 \text{ mm}$$

Example : 2.14

A cast iron hub of 50 mm outside diameter and 25 mm inside diameter is to be assembled on a 25 mm steel shaft with medium drive fit normal. Determine,

(i) Maximum and minimum interference

(ii) Maximum and minimum Contact pressure

(iii) Maximum and minimum tangential stress at contact surface

(iv) Maximum axial force required for the assembly if the hub length is 100 mm and $\mu = 0.12$

(v) Maximum torque. **[Bangalore University, Aug. 2002]**

Data :

$d_o = 50$ mm; $d_c = 25$ mm; $d_i = 0$ ($\because$ Solid shaft) ; $l = 100$ mm ; $\mu = 0.12$; Medium drive fit normal

Solution :

(i) Maximum and minimum interference

Form Table 11.6 (DDHB) for Medium drive fit normal, Combination of shaft and Hole is $H_7\, r_6$

Form Table 11.12 (DDHB) for r6 fit

Tolerance for 25 mm diameter shaft is $^{+41}_{+28}$

ie., Upper deviation = + 41 Microns

Lower deviation = + 28 Microns

∴ Shaft size is 25^{+41}_{+28}

ie., Maximum size of shaft $d_{s_{max}}$ = 25 + 0.041 = 25.041 mm

Minimum size of shaft $d_{s_{min}}$ = 25 + 0.028 = 25.028 mm

From 11.13 (DDHB) for H_7 fit,

Tolerance for 25 mm diameter hole is $^{+21}_{0}$

∴ Hole size is 25^{+21}_{0}

ie., Maximum size of hole $d_{h_{max}}$ = 25 + 0.021 = 25.021 mm

Minimum size of hole $d_{h_{min}}$ = 25 + 0 = 25 mm

∴ Maximum interference δ_{max} = 25.041 − 25 = 0.041 mm

Minimum interference δ_{min} = 25.028 − 25.021 = 0.007 mm

(ii) Maximum and minimum Contact pressure

From Table 2.10 (New DDHB),

For steel, Modulus of elasticity E = 206 GPa ; Poisson's ratio υ = 0.292

For cast iron, Modulus of elasticity E = 100 GPa ; Poisson's ratio υ = 0.211

For maximum Contact pressure,

$$\delta_{max} = \frac{p_{c\,max} \times d_{s\,max}}{E_s}\left[\frac{d_{s\,max}^2 + d_i^2}{d_{s\,max}^2 - d_i^2} - \upsilon_s\right] + \frac{p_{c\,max} \times d_{h\,min}}{E_h}\left[\frac{d_o^2 + d_{h\,min}^2}{d_o^2 - d_{h\,min}^2} + \upsilon_h\right] \quad \text{.... 11.5 a (DDHB)}$$

$$\text{ie., } 0.041 = \frac{p_{c\,max} \times 25.041}{206 \times 10^3}\left[\frac{25.041^2 + 0}{25.041^2 - 0} - 0.292\right] + \frac{p_{c\,max} \times 25}{100 \times 10^3}\left[\frac{50^2 + 25^2}{50^2 - 25^2} + 0.211\right]$$

∴ Maximum Contact pressure $p_{c_{max}}$ = 73.81 N/mm²

For minimum Contact pressure

$$\delta_{min} = \frac{p_{c\,min}\; d_{s\,min}}{E_s}\left[\frac{d_{s\,min}^2 + d_i^2}{d_{s\,min}^2 - d_i^2} - \upsilon_s\right] + \frac{p_{c\,min} \times d_{h\,max}}{E_h}\left[\frac{d_o^2 + d_{h\,max}^2}{d_o^2 - d_{h\,max}^2} + \upsilon_h\right]$$

$$\text{ie., } 0.007 = \frac{P_{c\,min} \times 25.028}{206 \times 10^3}\left[\frac{25.028^2 + 0}{25.028^2 - 0} - 0.292\right] + \frac{p_{c\,min} \times 25.021}{100 \times 10^3}\left[\frac{50^2 + 25.021^2}{50^2 - 25.021^2} + 0.211\right]$$

$\therefore$ Minimum contact pressure $p_{c\,min} = 12.529$ N/mm²

(iii) Maximum and Minimum tangential stress at contact surface

$$\text{Maximum tangential stress } \sigma_{\theta_{max}} = p_{c\,max}\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right) \quad \text{.... 11.13 (DDHB)}$$

$$= 73.81\left(\frac{50^2 + 25^2}{50^2 - 25^2}\right) = 123.02 \text{ N/mm}^2$$

$$\text{Minimum tangential stress } \sigma_{\theta_{min}} = p_{c\,min}\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right) = 12.529\left[\frac{50^2 + 25^2}{50^2 - 25^2}\right] = 123.02 \text{ N/mm}^2$$

(iv) Maximum axial force

$$F_{a\,max} = \pi\, d_c l \mu\, p_{c\,max} = \pi \times 25 \times 100 \times 0.12 \times 73.81 \quad \text{.... 11.22 (DDHB)}$$

$$= 69564.286 \text{ Nmm}$$

(v) Maximum torque

$$M_{t\,max} = \frac{\pi d_c^2 l \mu\, p_{c\,max}}{2} = F_{a_{max}} \cdot \frac{d_c}{2} \quad \text{.... 11.25 (DDHB)}$$

$$= 69564.286 \times \frac{25}{2} = 869.554 \times 10^3 \text{ Nmm}$$

$$= 869.554 \text{ Nm}$$

Example : 2.15

A high pressure cylinder consists of a steel tube with inner and outer diameters of 120 mm and 160 mm respectively. It is jacketed by an outer tube with an outer diameter of 200 mm. The tubes are assembled by a shrinking process in such a way that maximum principal stress induced is 36.45 N/mm². The shrink fit assembly is further subjected to an internal fluid pressure of 60 N/mm². Determine,

(i) Shrinkage pressure

(ii) Resultant tangential and radial stresses and plot the stress distribution.

Data :

d_i = 120 mm; d_c = 160 mm; d_o = 200 mm ; $\sigma_{\theta-oi}$ = 36.45 N/mm²; p_i = 60 N/mm²

Solution :

(i) Shrinkage pressure

Tangential stress at the inner surface of outer member (jacket) $\sigma_{0-oi} = p_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right)$ .... 11.13 (DDHB)

$$\text{ie., } 36.45 = p_c\left(\frac{200^2 + 160^2}{200^2 - 160^2}\right)$$

∴ Contact pressure or Shrinkage pressure $p_c = 8$ N/mm^2

(ii) Stresses due to internal fluid pressure (ie., $p_i = 60$ N/mm^2)

If the compound cylinder is subjected to an internal fluid pressure p_i and external pressure $p_o = 0$, then the general expression for tangential stress according to Lame's equation is,

$$\sigma_0 = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{4r^2}\right) \qquad \text{.... 7.19 (DDHB)}$$

Tangential stress at the outer surface of outer cylinder (ie., jacket) due to internal fluid pressure p_i

$$\sigma'_{0_{oo}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{d_o^2}\right) = \frac{2p_i d_i^2}{d_o^2 - d_i^2} \quad \left(\because r = \frac{d_o}{2}\right)$$

$$= \frac{2 \times 60 \times 120^2}{200^2 - 120^2} = 67.5 \text{ N/mm}^2 \text{ (Tensile)}$$

Tangential stress at the inner surface of outer cylinder (ie., jacket) or Outer surface of inner member (ie., tube)

$$\sigma'_{0_{oi}} = \sigma'_{0_{io}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{d_c^2}\right)\left(\because r = \frac{dc}{2}\right)$$

$$= \frac{60 \times 120^2}{200^2 - 120^2}\left(1 + \frac{200^2}{160^2}\right) = 86.5 \text{ N/mm}^2 \text{ (Tensile)}$$

Tangential stress due to internal fluid pressure p_i at the inner surface of inner member

$$\sigma'_{0_{ii}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{d_i^2}\right)\left(\because r = \frac{d_i}{2}\right)$$

$$= \frac{60 \times 120^2}{200^2 - 120^2}\left(1 + \frac{200^2}{120^2}\right) = 127.5 \text{ N/mm}^2 \text{ (Tensile)}$$

Tangential stress distribution due to internal fluid pressure is shown in Fig. 2.13 (a)

(b) Radial stress distribution due to internal fluid pressure

If the compound cylinder is subjected to an internal fluid pressure p_i and external pressure $p_o = 0$, then the general expression for radial stress according to Lame's equation is

$$\sigma_r = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{4r^2}\right) \quad \text{.... 7.20 (DDHB)}$$

Radial stress at the outer surface of outer cylinder (ie., jacket) due to internal fluid pressure p_i

$$\sigma'_{r_{oo}} = \frac{p_i d_i^2}{\left(d_o^2 - d_i^2\right)}\left(1 - \frac{d_o^2}{d_o^2}\right) = 0 \left(\because r = \frac{d_o}{2}\right)$$

Radial stress at the inner surface of outer cylinder (ie., jacket) or Outer surface of inner member (ie., tube) due to internal fluid pressure p_i

$$\sigma'_{r_{oi}} = \sigma'_{r_{io}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{d_c^2}\right)\left(\because r = \frac{d_c}{2}\right)$$

$$= \frac{60 \times 120^2}{\left(200^2 - 120^2\right)}\left(1 - \frac{200^2}{160^2}\right) = -33.75 \text{ N/mm}^2 \text{ (Compressive)}$$

Radial stress at the inner surface of inner member due to internal fluid pressure p_i

$$\sigma'_{r_{ii}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 - \frac{d_o^2}{d_i^2}\right)\left(\because r = \frac{d_i}{2}\right)$$

$$= -p_i = -60 \text{ N/mm}^2 \text{ (Compressive)}$$

Radial stress distribution due to internal fluid pressure is shown in Fig. 2.14 (a)

(iii) Stresses due to shrink fit

(a) Tangential stress distribution due to shrink fit

Tangential stress at outside diameter of outer member (ie., jacket)

$$\sigma_{\theta-oo} = \frac{2p_c d_c^2}{d_o^2 - d_c^2} = \frac{2 \times 8 \times 160^2}{200^2 - 160^2} = 28.44 \text{ N/mm}^2 \text{ (Tensile)} \quad \text{.... 11.12 (DDHB)}$$

Tangential stress at the inside diameter of outer member (ie., jacket)

$$\sigma_{\theta-oi} = p_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right) = 8\left(\frac{200^2 + 160^2}{200^2 - 160^2}\right) = 36.44 \text{ N/mm}^2 \text{ (Tensile)} \quad \text{.... 11.13 (DDHB)}$$

Tangential stress at outside diameter of inner cylinder (ie., tube)

$$\sigma_{\theta-io} = -p_c\left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2}\right) = -8\left(\frac{160^2 + 120^2}{160^2 - 120^2}\right) = -28.57 \text{ N/mm}^2 \text{ (Compressive)} \quad \text{.... 11.14 (DDHB)}$$

Tangential stress at inside diameter of inner cylinder (ie., tube)

$$\sigma_{\theta-ii} = -\frac{2p_c d_c^2}{d_c^2 - d_i^2} = -\frac{2 \times 8 \times 160^2}{160^2 - 120^2} = -36.57 \text{ N/mm}^2 \text{ (Compressive)} \quad .... \text{ 11.15 (DDHB)}$$

Tangential stress distribution due to shrinkage fitting is shown in Fig. 2.13 (b)

(b) Radial stress distribution due to shrink fit ;

Radial stress at outside diameter of outer member (ie., jacket) $\sigma_{r-oo} = 0$ .... 11.16 (DDHB)

Radial stress at inside diameter of outer member (ie., jacket) $\sigma_{r-oi} = -p_c = -8$ N/mm² (Compressive) .... 11.17 (DDHB)

Radial stress at outside diameter of inner member (ie., tube) $\sigma_{r-io} = -p_c = -8$ N/mm² (Compressive) .... 11.18 (DDHB)

Radial stress at inside diameter of inner member (ie., tube) $\sigma_{r-ii} = 0$ .... 11.19 (DDHB)

Radial stress distribution due to internal shrinkage fitting is shown in Fig. 2.14 (b).

(iv) Resultant stress distribution

(a) Resultant tangential stress distribution due to internal fluid pressure and shrink fit

Resultant tangential stress at outside diameter of outer member (ie., jacket)

$$= \sigma'_{\theta oo} + \sigma_{\theta-oo} = 67.5 + 28.44 = 95.94 \text{ N/mm}^2 \text{(Tensile)}$$

Resultant tangential stress at inside diameter of outer member (ie., jacket)

$$= \sigma'_{\theta\, oi} + \sigma_{\theta-oi} = 86.5 + 36.44 = 122.94 \text{ N/mm}^2 \text{ (Tensile)}$$

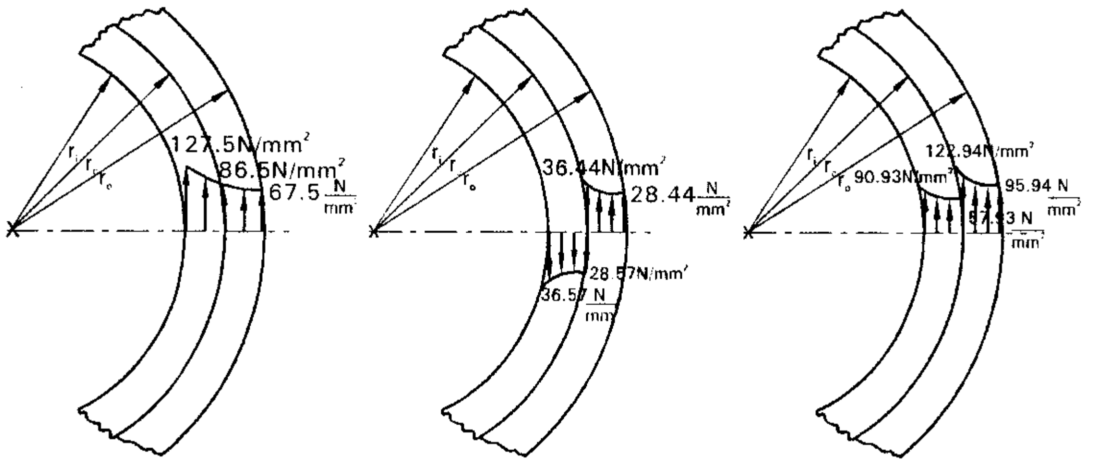
Resultant tangential stress at outside diameter of inner member (ie tube)

$$= \sigma'_{\theta\, io} + \sigma_{\theta-io} = 86.5 - 28.57 = 57.93 \text{ N/mm}^2 \text{ (Tensile)}$$

Resultant tangential stress at inside diameter of inner member (ie., tube)

$$= \sigma'_{\theta\, ii} + \sigma'_{\theta-ii} = 127.5 - 36.57 = 90.93 \text{ N/mm}^2 \text{ (Tensile)}$$

Resultant tangential stress distribution due to internal fluid pressure and shrinkage fitting is shown in Fig. 2.13 (c).



a) Tangential stress distribution due to internal fluid pressure

b) Tangential stress distribution due to shrinkage fitting

c) Resultant tangential stress due to internal fluid pressure distribution and shrinkage fitting

Fig. 2.13

(b) Resultant radial stress distribution due to internal fluid pressure and shrink fit

Resultant radial stress at outside diameter of outer member (ie., jacket)

$$= \sigma'_{roo} + \sigma_{r-oo} = 0 + 0 = 0$$

Resultant radial stress at inside diameter of outer member (ie., jacket)

$$= \sigma'_{roi} + \sigma_{r-oi} = -37.75-8 = -41.75 \text{ N/mm}^2 \text{ (Compressive)}$$

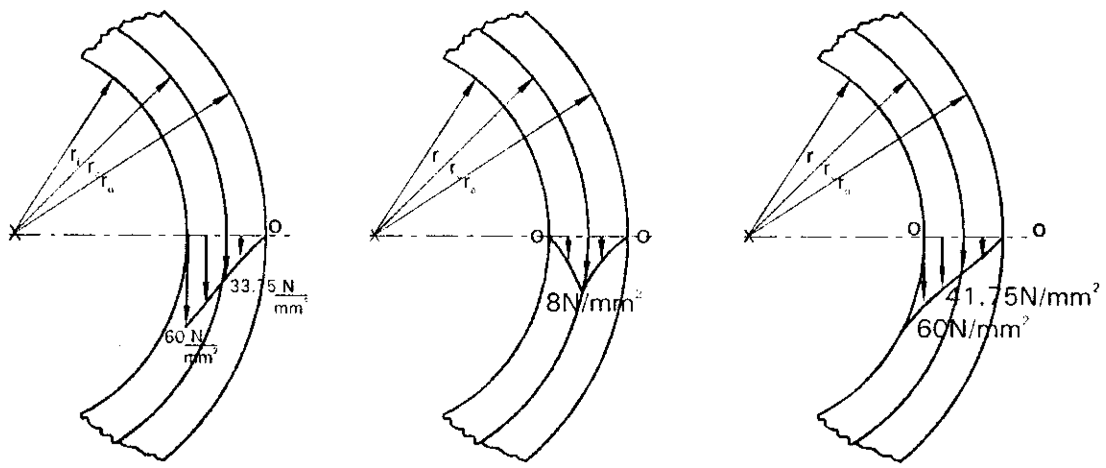
Resultant radial stress at outside diameter of inner member (ie., tube)

$$= \sigma'_{rio} + \sigma_{r-io} = -33.75 - 8 = -41.75 \text{ N/mm}^2 \text{ (Compressive)}$$

Resultant radial stress at inside diameter of inner member (ie., tube)

$$= \sigma'_{rii} + \sigma_{r-ii} = -60 + 0 = -60 \text{ N/mm}^2 \text{ (Compressive)}$$

Resultant radial stress distribution due to internal fluid pressure and shrinkage fitting is shown in Fig.2.14 (c).



a) Radial stress distribution due to internal fluid pressure

b) Radial stress distribution due to shrinkage fitting

c) Resultant radial stress distribution due to internal fluid pressure and shrinkage fitting

Fig. 2.14

Example : 2.16

A steel tube with inner and outer diameters of 50 mm and 75 mm respectively is jacketed by an outer steel tube with an outer diameter of 100 mm. The compound tube is subjected to an internal pressure of 35 MPa. The shrinkage allowance is such that the maximum tangential stress in each tube has same magnitude. Find,

(i) Shrinkage pressure

(ii) Original dimensions of tubes

Also plot the distribution of tangential stresses.

Data :

$d_i = 50$ mm ; $d_c = 75$ mm ; $d_o = 100$ mm ; $p_i = 35$ MPa

Solution :

Tangential stress distribution due to internal fluid pressure p_i

General expression for tangential stress according to Lame's equation is,

$$\sigma_\theta = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{4r^2}\right) \text{ when } p_o = 0 \qquad \text{.... 7.19 (DDHB)}$$

Tangential stress at the outer surface of outer cylinder (ie., jacket) due to internal fluid pressure pi

$$\sigma'_{\theta_{oo}} = \frac{p_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{d_o^2}\right) = \frac{2p_i d_i^2}{d_o^2 - d_i^2} \qquad \left(\because r = \frac{d_o}{2}\right)$$

$$= \frac{2\times35\times50^2}{100^2-50^2} = 23.33 \text{ N/mm}^2 \text{ (Tensile)}$$

Tangential stress at the inner surface of outer cylinder (ie., jacket) or Outer surface of inner member (ie., tube) due to internal fluid pressure

$$\sigma'_{\theta_{oi}} = \sigma'_{\theta_{io}} = \frac{p_i d_i^2}{d_o^2-d_i^2}\left(1+\frac{d_o^2}{d_c^2}\right) \left(\because r=\frac{d_c}{2}\right)$$

$$= \frac{35\times50^2}{100^2-50^2}\left(1+\frac{100^2}{75^2}\right) = 32.41 \text{ N/mm}^2 \text{ (Tensile)}$$

Tangential stress at the inner surface inner member (ie., tube) due to internal fluid pressure

$$\sigma'_{\theta_{ii}} = \frac{p_i d_i^2}{d_o^2-d_i^2}\left(1+\frac{d_o^2}{d_i^2}\right)\left(\because r=\frac{d_i}{2}\right)$$

$$= \frac{35\times50^2}{100^2-50^2}\left(1+\frac{100^2}{50^2}\right) = 58.33 \text{ N/mm}^2 \text{ (Tensile)}$$

Tangential stress distribution due to shrinkage fitting

Tangential stress at outside diameter of outer member (ie., jacket)

$$\sigma_{\theta-oo} = \frac{2p_c d_c^2}{d_o^2-d_c^2} = \frac{2p_c\times75^2}{100^2-75^2} = 2.57\ p_c \text{ (Tensile)} \quad \text{.... 11.12 (DDHB)}$$

Tangential stress at inside diameter of outer member (ie., jacket)

$$\sigma_{\theta-oi} = p_c\left(\frac{d_o^2+d_c^2}{d_o^2-d_c^2}\right) = p_c\left(\frac{100^2+75^2}{100^2-75^2}\right) = 3.57\ p_c \text{ (Tensile)}$$

.... 11.13 (DDHB)

Tangential stress at outside diameter of inner member (ie., tube)

$$\sigma_{\theta-io} = -p_c\left(\frac{d_c^2+d_i^2}{d_c^2-d_i^2}\right) = -p_c\left(\frac{75^2+50^2}{75^2-50^2}\right) = -2.6\ p_c \text{ (Compressive)}$$

.... 11.15 (DDHB)\

Tangential stress at inside diameter of inner member (ie., tube)

$$\sigma_{\theta-ii} = \frac{-2p_c d_c^2}{d_c^2-d_i^2} = \frac{-2p_c\times75^2}{(75^2-50^2)} = -3.6\ p_c \text{ (Compressive).... 11.15 (DDHB)}$$

(i) Shrinkage pressure

As the maximum tangential stress in each tube has same magnitude, equating stresses at the inner surfaces of tube and jacket,

$$\text{ie., } \sigma'_{\theta_{ii}} + \sigma_{\theta-ii} = \sigma'_{\theta_{oi}} + \sigma_{\theta_{-oi}}$$

$$\text{ie., } 58.33 - 3.6\, p_c = 32.41 + 3.57\, p_c$$

$\therefore$ Shrinkage pressure or Contact pressure $p_c = 3.615$ N/mm^2

(ii) Original dimensions of tubes

From Table 2.10 (New DDHB) Vol.I for steel

$$E = 206 \text{ GPa} = 206 \times 10^3 \text{ N/mm}^2 \text{ ; } \upsilon = 0.292$$

Change in diameter of inner member $\Delta d_i = -\dfrac{p_c d_c}{E}\left(\dfrac{d_c^2 + d_i^2}{d_c^2 - d_i^2} - \upsilon\right)$ .... 11.2 (DDHB)

$$= -\frac{3.615 \times 75}{206 \times 10^3}\left[\frac{75^2 + 50^2}{75^2 - 50^2} - 0.292\right] = -3.038 \times 10^{-3} \text{ mm}$$

Change in diameter of outer member $\Delta d_o = \dfrac{p_c d_c}{E}\left(\dfrac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + \upsilon\right)$ .... 11.3 (DDHB)

$$= \frac{3.615 \times 75}{206 \times 10^3}\left[\frac{100^2 + 75^2}{100^2 - 75^2} + 0.292\right] = 5.085 \times 10^{-3} \text{ mm}$$

$\therefore$ Original outside diameter of inner member (ie., tube) $d_s = d_c + \Delta d_i = 75 + 3.038 \times 10^{-3} = 75.003038$ mm

Original inside diameter of outer member (ie., jacket) $d_h = d_c - \Delta d_o = 75 - 5.085 \times 10^{-3} = 74.994915$ mm

$$\text{Total interference } \delta = \Delta d_i + \Delta d_o = 3.038 \times 10^{-3} + 5.085 \times 10^{-3} = 8.123 \times 10^{-3} \text{ mm}$$

ie., If we keep outside diameter of inner member exactly as 75 mm, then inside diameter of outer member (ie., jacket) $\approx 75 - 8.123 \times 10^{-3} = 74.9919$ mm

(iii) Resultant tangential stress distribution and the sketch

Tangential stress due to shrink fit at outside diameter of outer member (ie., jacket)

$$\sigma_{\theta-oo} = 2.57\, p_c = 2.57 \times 3.615 = 9.29 \text{ N/mm}^2 \text{ (Tensile)}$$

Tangential stress due to shrink fit at inside diameter of outer member (ie., jacket)

$$\sigma_{\theta-oi} = 3.57\, p_c = 3.57 \times 3.615 = 12.91 \text{ N/mm}^2 \text{ (Tensile)}$$

Tangential stress due to shrink fit at out side diameter of inner member (ie., tube)

$$\sigma_{\theta-io} = -2.6\, p_c = -2.6 \times 3.615 = -9.4 \text{ N/mm}^2 \text{ (Compressive)}$$

Tangential stress due to shrink fit at inside diameter of inner member (ie., tube)

$$\sigma_{\theta-ii} = -3.6\,p_c = -3.6 \times 3.615 = -13.01 \text{ N/mm}^2 \text{ (Compressive)}$$

Resultant tangential stress due to internal fluid pressure and shrink fit at outside diameter of outer member (ie., jacket) = $\sigma'_{\theta_{oo}} + \sigma_{\theta-oo} = 23.33 + 9.29 = 32.62$ N/mm^2 (Tensile)

Resultant tangential stress due to internal fluid pressure and stress fit at inside diameter of outer member (ie., jacket) = $\sigma'_{\theta_{oi}} + \sigma_{\theta-oi} = 32.41 + 12.91 = 45.32$ N/mm^2 (Tensile)

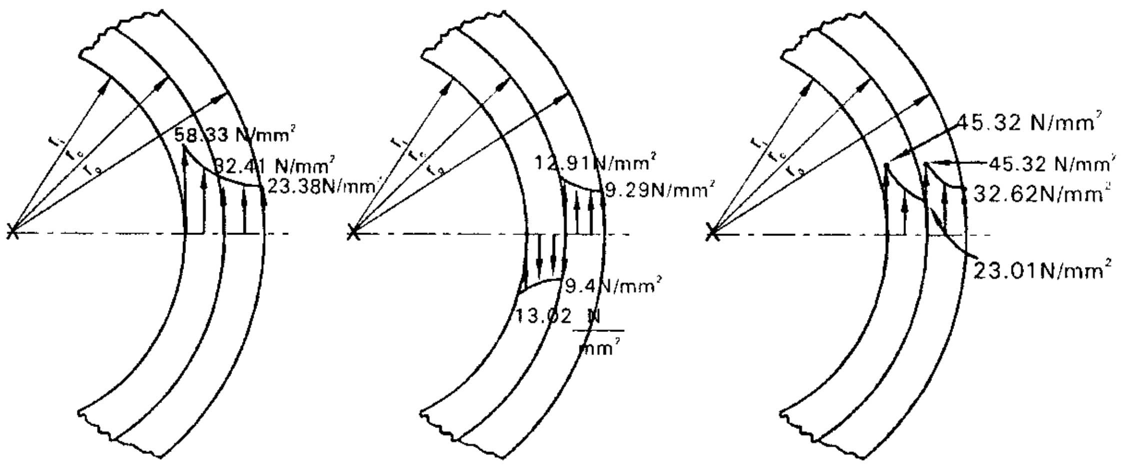
Resultant tangential stress due to internal fluid pressure and shrink fit at outside diameter of inner member (ie., tube) = $\sigma'_{\theta_{io}} + \sigma_{\theta-io} = 32.41 - 9.4 = 23.01$ N/mm^2 (Tensile)

Resultant tangential stress due to internal fluid pressure and shrink fit at inside diameter of inner member (ie., tube) = $\sigma'_{\theta_{ii}} + \sigma_{\theta-ii} = 58.33 - 13.01 = 45.32$ N/mm^2 (Tensile)

Tangential stress distribution due to internal fluid pressure is shown in Fig. 2.15 (a)

Tangential stress distribution due to shrinkage fitting is shown in Fig. 2.15 (b)

Resultant tangential stress distribution due to internal fluid pressure and shrinkage fitting is shown in Fig. 2.15 (c)



(a) (b) (c)

Fig. 2.15

2.12 CYLINDER HEADS AND COVER PLATES

The heads of cylindrical pressure vessels and the sides of square or rectangular tanks usually consists of flat plates or slightly dished plates. These plates can either be cast integrally with the cylinder walls or fixed to them by means of rivets, welds or bolts. Depending upon the type of connection between the head and the cylindrical wall, it is classified as

(i) Freely supported and

(ii) Rigidly fixed.

Depending upon the nature of load, it is classified as

(i) Uniformly distributed load and

(ii) Concentrated load.

The design of flat plates forming the heads depend upon the following two factors

(i) Type of connection with the supporting members

(ii) Nature of load

2.12.1 Circular flat plate with uniformly distributed load

The thickness of a plate with diameter 'd' supported freely at the circumference and subjected to a pressure 'p' distributed uniformly over the total area

$$h = k_1 d \sqrt{\frac{p}{\sigma_d}} \qquad \text{----- 8.1 (DDHB)}$$

where σ_d = Allowable or Permissible design stress

d = Diameter of plate

p = Pressure

k_1 = Coefficient. It depends upon the material of the plate and the method of holding the edges Table 8.3 (DDHB)

$$\text{Maximum deflection } y = k_2 d^4 \frac{p}{Eh^3} \qquad \text{----- 8.2 (DDHB)}$$

Coefficient k_2 can be obtained from Table 8.3 (DDHB)

Also the values of h and y_{max} can be obtained from Table 8.2 (DDHB)

From Table 8.2 (DDHB) Maximum allowable stress

$$\sigma_{max} = \sigma_d = \frac{3F(3m+1)}{8\pi m h^2}$$

where F = Total load $\pi r_0^2 p$; 1/m = Poission's ratio ; h = Thickness of plate

The thickness of a plate with diameter d edges rigidly fixed and subjected to a pressure 'p' distributed uniformly over the total area

$$h = \sqrt{\frac{3F}{4\pi\sigma_r}} \qquad \text{----- Table 8.2 (DDHB)}$$

2.12.2 Circular Plates loaded Centrally

The thickness of a flat cast iron circular plate with diameter d and subjected to a load F distributed uniformly over an area $\frac{\pi}{4}d_o^2$

$$h = 1.2\sqrt{\left(1-\frac{0.67d_o}{d}\right)\frac{F}{\sigma_d}} \text{ for freely supported plate} \quad \text{----- 8.3 (DDHB)}$$

$$h = 0.65\sqrt{\frac{F}{\sigma_b}\log_e\left(\frac{d}{d_o}\right)} \text{ for fixed plate} \quad \text{----- 8.5 (DDHB)}$$

Also the values of y and h can be obtained from Table 8.2 (DDHB)

2.12.3 Rectangular flat plates

Thickness of a rectangular plate according to Grashof and Back

$$h = abk_3\sqrt{\frac{p}{\sigma_d\left(a^2+b^2\right)}} \quad \text{----- 8.7 (DDHB)}$$

where a = Length of plate

b = Width of plate

k_3 = Coefficient, from Table 8.3 (DDHB)

Thickness of rectangular plates can also be obtained using the formulae from Table 8.2 (DDHB)

The other formulae commonly used are

$$h = ab\sqrt{\frac{p}{2\sigma_d\left(a^2+b^2\right)}} \text{ For freely supported edges}$$

$$= ab\sqrt{\frac{p}{3\sigma_d\left(a^2+b^2\right)}} \text{ For fixed plates}$$

Thickness of a rectangular plate on which a concentrated load F acts at the intersection diagonals

$$h = k_4\sqrt{\frac{abF}{\sigma_d\left(a^2+b^2\right)}} \quad \text{----- 8.8 (DDHB)}$$

where k_4 = Coefficient, from Table 8.3 (DDHB)

2.12.4 Elliptical cover plate

Thickness of uniformly loaded elliptical cover plate

$$h = ab\,k_5\sqrt{\frac{p}{\sigma_d\left(a^2+b^2\right)}} \qquad \text{----- 8.9 (DDHB)}$$

where a = Major axis

b = Minor axis

k_5 = Coefficient, from Table 8.3 (DDHB)

2.12.5 Dished Heads

Many cylinders are provided with semispherical heads as shown in Fig 2.16. The thickness of a dished head that is riveted or welded to cylindrical shell according to ASME Boiler code

$$h = \frac{8.33pR}{2\sigma_u} \qquad \text{----- 8.39 (DDHB)}$$

where R = Inside radius of curvature of head

σ_u = Ultimate strength of the material

d = Shell diameter

R should be greater than shell diameter. If $R < 0.8$ d, then take $R = 0.8$ d. If there is an opening in the head, the thickness of the head should be increased by 15% but should not be less than 3.2 mm

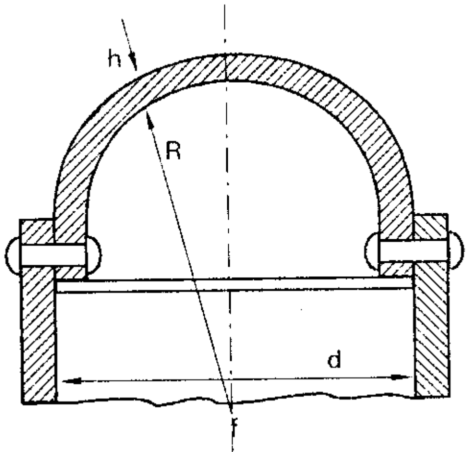


Fig 2.16

2.12.6 Standard Semi-ellipsoidal head

When the dished plate is fixed integrally or welded to the cylinder and is of standard semi ellipsoidal shape as shown in Fig 2.17, then

$$\text{Thickness of head } h = \frac{p_i d_i}{2\sigma_0\eta - 0.2p_i} + CA$$

where σ_0 = Allowable tensile stress (Tangential or Hoopstress)

η = Efficiency

d_i = Shell diameter

p_i = Allowable maximum pressure

CA = Corrosion allowance

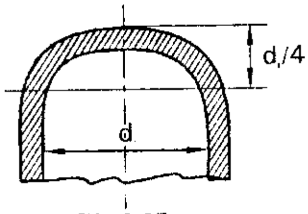


Fig 2.17

2.12.7 Hemispherical head

When the dished plate is fixed integrally or welded to the cylinder and is of hemispherical shape as shown in Fig 2.18, then

Thickness of head $h = \frac{p_i d_i}{4\sigma_0 \eta - 0.4 p_i} + CA$

where CA = corrosion allowance

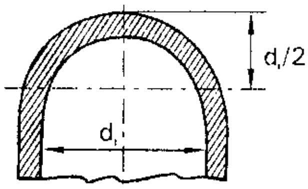


Fig 2.18

2.12.8 Conical head

When the dished plate is fixed integrally or welded to the cylinder and is of conical shape as shown in Fig 2.19, then

Thickness of head $h = \frac{p_i d_i}{(2\sigma_0 \eta - 1.2 p_i)} + CA$

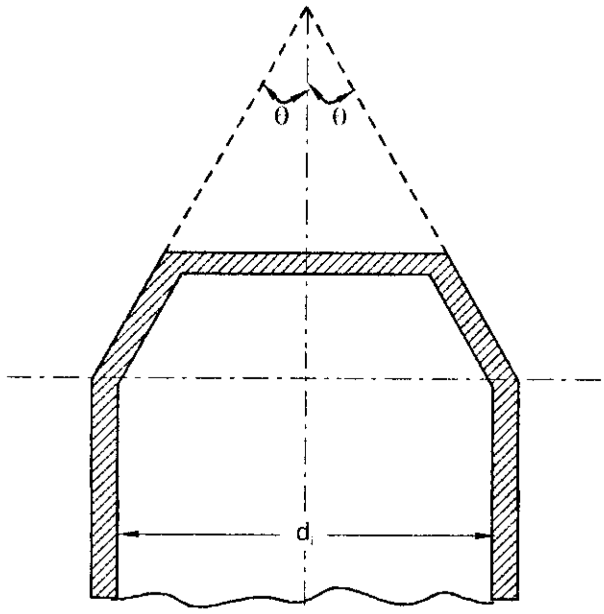


Fig 2.19

2.12.9 Unstayed flat heads

Fig 2.20 shows the construction of various types unstayed steel heads, coverplates, blind flanges etc. The minimum thickness of unstayed steel flat head or cover plate with uniformly distributed load is given by

$$h = d\sqrt{\frac{c_3 p}{\sigma_d}} \qquad \text{----- 8.40 (DDHB)}$$

where d = Shell diameter

c_3 = Coefficient, from Table 8.1 (DDHB)

p = Allowable or Maximum pressure

σ_d = Allowable design stress, from Table 8.9 (DDHB)

According to Grashof's formula for stayed flat plates with uniformly distributed load,

$$\text{Maximum stress } \sigma = 0.2275\,\frac{a^2 p}{h^2} \qquad \text{----- 8.42 (DDHB)}$$

2.12.10 Torispherical head

When the dished plate is fixed integrally or welded to the cylinder and is of Torispherical shape as shown in Fig 2.21, then

Thickness of head $h = \frac{0.855 p_i L}{\sigma_0 \eta - 0.1 p_i} + CA$ where L is the inside crown radius. The knuckle radius r_i is taken as 6% of crown radius. The crown radius L should not be greater than the outside diameter (d_o) of the cylindrical shell. Length of straight portion l_s = 3 h or 20 mm (which ever is more)

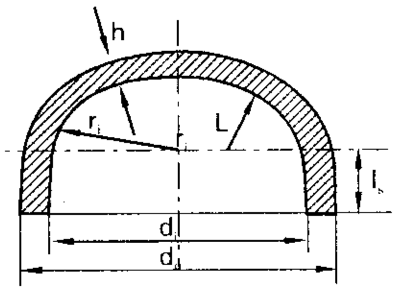


Fig 2.21

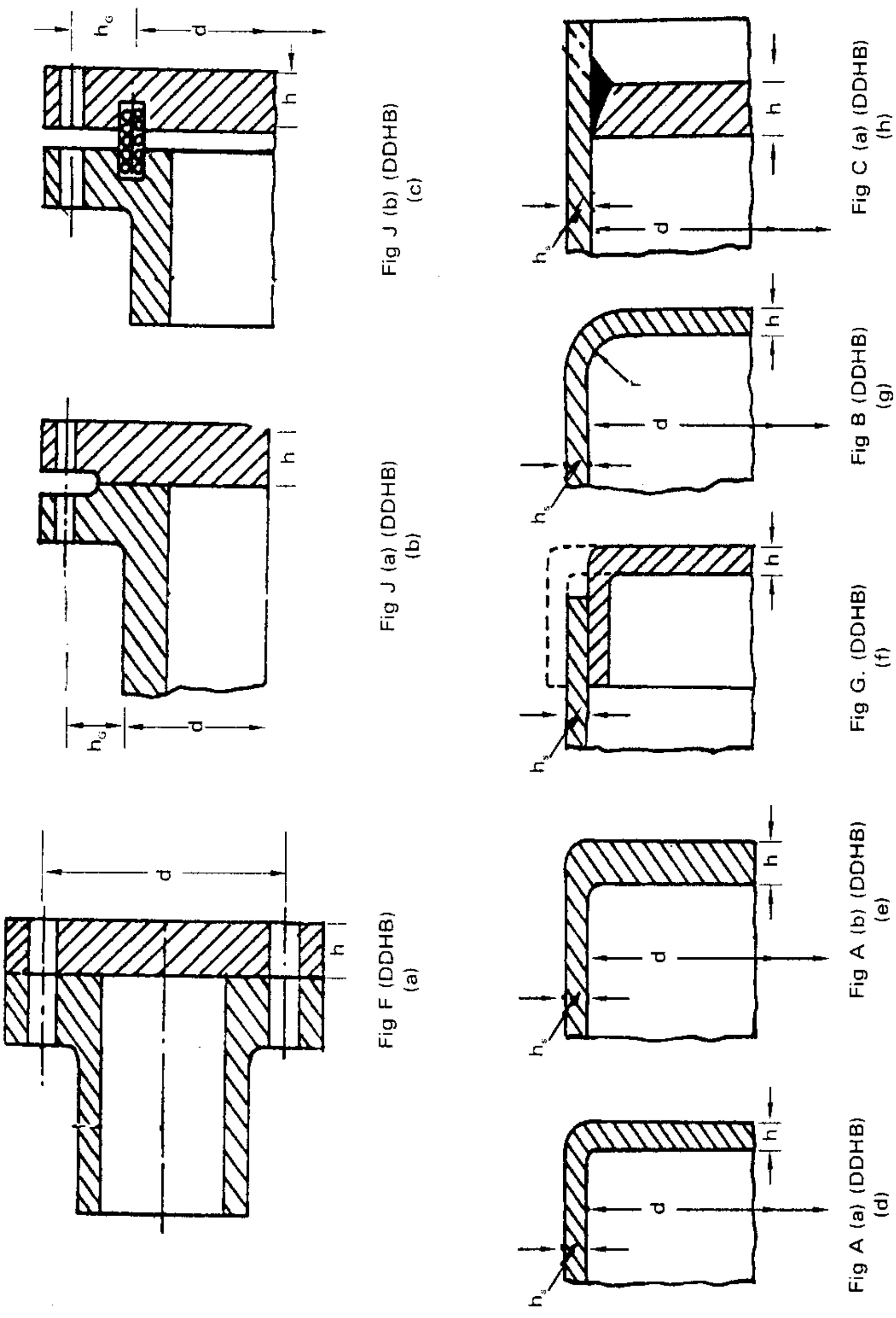


Fig 2.20 (Fig 8.1 DDHB)

Example : 2.17

A cast iron thick cylinder of internal diameter 150 mm is subjected to an internal pressure of 12 N/mm^2. The allowable working stress for the cast iron may be taken as 20 N/mm^2 Determine

(i) Thickness of cylinder wall

(ii) Thickness of the circular flat cylinder head cast integral with the cylinder walls.

Data :

$p = 12\ N/mm^2$; $\sigma_d = 20\ N/mm^2$; $d = 150$ mm

Solution :

(i) Thickness of cylinder wall

According to Lame's equation thickness of CI thick cylinder wall

$$h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta + p_i}{\sigma_\theta - p_i}} - 1\right] = \frac{150}{2}\left[\sqrt{\frac{20+12}{20-12}} - 1\right] = 75 \text{ mm} \qquad \text{----- 7.24 (DDHB)}$$

(ii) Thickness of flat circular cylinder head

$$h = k_1 d\sqrt{\frac{p}{\sigma_d}} \qquad \text{----- 8.1 (DDHB)}$$

From Table 8.3 (DDHB) for cast iron fixed edges ($\because$ Cast integral with cylinder walls)

Coefficient $k_1 = 0.44$

$\therefore$ Thickness of head $h = 0.44 \times 150\sqrt{\frac{12}{20}} = 51.12$ mm

From Table 8.2 (DDHB) for flat circular head with distributed load and edge fixed

Maximum allowable stress $\sigma_d = \frac{3F}{4\pi h^2}$ where $F = \pi r_o^2 p$

i.e., $20 = \frac{3 \times \pi \times 75^2 \times 12}{4\pi h^2}$

$\therefore$ Thickness of head h = 50.31 mm

Adopt the bigger value i.e., h = 51.12 mm, Take, Thickness of head h = 55 mm

Example : 2.18

A cylinder is provided with a head of flat circular steel plate of 500 mm diameter and is supported around the edge. It is subjected to a uniform pressure of 5 N/mm^2. The allowable working stress for the material is 70 N/mm^2 and Poisson's ratio is 0.3. Determine the

(i) Thickness of thick cylinder wall

(ii) Thickness of the circular flat cylinder head.

Data :

$$d = 500\ mm;\quad p = 5\ N/mm^2;\quad \sigma_d = 70\ N/mm^2;\quad \frac{1}{m} = 0.3$$

Solution :

(i) Thickness of cylinder wall

According to Lame's equation thickness of thick steel cylinder wall

$$h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta}{\sigma_\theta - 2p}} - 1\right] = \frac{500}{2}\left[\sqrt{\frac{70}{70 - 2\times 5}} - 1\right] = 20\ mm \qquad \text{----- 7.25 (DDHB)}$$

(ii) Thickness of flat circular cylinder head

$$h = k_1 d\sqrt{\frac{p}{\sigma_d}} \qquad \text{----- 8.1 (DDHB)}$$

From Table 8.3 (DDHB) for steel supported at edges coefficient $k_1 = 0.42$

$$\therefore h = 0.42 \times 500\sqrt{\frac{5}{70}} = 56.12\ mm$$

From Table 8.2 (DDHB) for flat circular head with distributed load and supported edges (i.e., free)

$$\text{Maximum allowable stress } \sigma_d = \frac{3F(3m+1)}{8\pi m h^2} \quad \text{where } F = \pi r_o^2 p$$

$$\text{i.e.,}\quad 70 = \frac{3\times\pi\times 250^2\times 5\left(3\times\frac{1}{0.3}+1\right)}{8\times\pi\times\frac{1}{0.3}\times h^2}$$

$$\therefore h = 74.32\ mm$$

Adopt the bigger value, i.e., h = 74.32 mm

Take, thickness of head h = 75 mm

Example : 2.19

A fusion welded thin cylindrical shell of internal diameter 200 mm is filled with ammonia gas, under pressure 5 N/mm². The ends of the cylindrical shell are closed by dished head with the inside covex. The radius of curvature of the head is 200mm. The material of the cylindrical shell and head is Fe 360 steel and factor of safety 5. Determine,

(i) Thickness of cylindrical shell and

(ii) Thickness of head

Data :

d = 200 mm; p = 5 N/mm²; R = 200 mm; Material – Fe 360 steel; FOS = 5;

Solution :

From Table 1.9 (New DDHB) for Fe 360 steel $\sigma_{u_t} = 360$ N/mm²

$$\therefore \sigma_d = \sigma_0 = \frac{\sigma_{u_t}}{FOS} = \frac{360}{5} = 72 \text{ N/mm}^2$$

(i) Thickness of cylindrical shell

Maximum thickness of shell exclusive of corrosion allowance

$$h = \frac{pd_i}{2\sigma_\theta \eta - p} \qquad \text{----- 8.10 (DDHB)}$$

From Table 8.4 (DDHB), for double welded butt joint

Joint efficiency factor $\eta = 0.85$

$$\therefore h = \frac{5 \times 200}{2 \times 72 \times 0.85 - 5} = 8.52 \text{ mm}$$

Considering corrosion allowance, take the thickness of cylindrical shell h = 10 mm.

(ii) Thickness of dished head

Thickness of a dished head that is riveted or welded to cylindrical shell $h = \frac{8.33pR}{2\sigma_u}$ ----- 8.39 (DDHB)

$$= \frac{8.33 \times 5 \times 200}{2 \times 360} = 11.57 \text{ mm} \cong 12 \text{ mm}$$

Example : 2.20

A fusion welded thin cylindrical shell of internal diameter 2000 mm is subjected to an internal pressure of 1 N/mm². The yield strength of the material of the shell is 250 N/mm² with a factor of safety of 2.5. The ends of the cylindrical shell are closed by torispherical heads with a crown radius of 1.6 m and the corrosion allowance is 3 mm. Determine,

(i) Thickness of cylindrical shell and

(ii) Thickness of head

Data :

d = 2000 mm; p = 1 N/mm²; σ_{y_t} = 250 N/mm²; FOS = 2.5; L = 1.6 m = 1600 mm; CA = 3 mm

Solution :

(i) Thickness of cylindrical shell

Maximum thickness of cylindrical shell inclusive of corrosion allowance

$$h = \frac{pd_i}{2\sigma_\theta \eta - p} + CA \text{ where CA = corrosion allowance} \qquad \text{----- 8.10 (DDHB)}$$

From Table 8.4 (DDHB), for double welded butt joint

Joint efficiency factor $\eta = 0.85$

Permissible or Allowable design stress $\sigma_d = \sigma_0 = \frac{\sigma_{yt}}{FOS} = \frac{250}{2.5} = 100 \text{ N/mm}^2$

$$\therefore h = \frac{1 \times 2000}{2 \times 100 \times 0.85 - 1} + 3 = 14.76 \text{ mm} \cong 15 \text{ mm}$$

$\therefore$ Thickness of cylindrical shell = 15 mm

(ii) Thickness of head

Thickness of torishperical head

$$h = \frac{0.885 p_i L}{\sigma_0 \eta - 0.1 p_i} + CA = \frac{0.885 \times 1 \times 1600}{100 \times 0.85 - 0.1 \times 1} + 3 = 19.68 \cong 20 \text{ mm}$$

$\therefore$ Thickness of torispherical head h = 20 mm

Example : 2.21

An engine's chest is covered by a flat rectangular head of 200 mm × 300 mm dimension. The plate is made of grey cast iron FG 150 material, supported at the edges and is subjected to a uniform pressure of 1.5 N/mm². Determine the thickness of the head for a factor of safety of 5.

Data :

a = 300 mm; b = 200 mm; p = 1.5 N/mm²; FOS = 5 ; Material for the head - Grey cast iron FG 150

Solution :

From Table 1.4 (New DDHB) for grey cast iron FG 150

$$\sigma_{ut} = 150 \text{ N/mm}^2$$

$\therefore$ Allowable design stress $\sigma_d = \frac{\sigma_{ut}}{FOS} = \frac{150}{5} = 30 \text{ N/mm}^2$

Thickness of a rectangular plate subjected to uniform load according to Grashof's and Back

$$h = ab\, k_3 \sqrt{\frac{p}{\sigma_d (a^2 + b^2)}} \qquad \text{----- 8.7 (DDHB)}$$

where a = Length of plate = 300 mm

b = Breadth of plate = 200 mm

From Table 8.3 (DDHB) for cast iron supported, Coefficient $k_3 = 0.75$

$$\therefore h = 300 \times 200 \times 0.75 \sqrt{\frac{1.5}{30(300^2 + 200^2)}} = 27.9 \cong 25 \text{ mm}$$

From Table 8.2 (DDHB) for rectangular plate, all edges supported

$$\text{Maximum allowable stress } \sigma_d = \frac{0.75b^2p}{h^2\left(1+1.61\times\frac{b^3}{a^3}\right)}$$

$$\text{i.e., } 30 = \frac{0.75\times 200^2\times 1.5}{h^2\left(1+1.61\times\frac{200^3}{300^3}\right)}$$

$$\therefore h = 31.86 \cong 32 \text{ mm}$$

Adopt the larger value.

∴ Thickness of rectangular plate head h = 32 mm.

Example : 2.22

The following data refers a diesel engine.
Inside cylinder diameter = 150 mm
Explosion pressure = 5 N/mm²
Material for the cylinder and head = Grey CI FG 150
Factor of safety = 5
Design (i) cylinder and (ii) Head.

Solution :

(i) **Design of cylinder**

From Table 1.4 (New DDHB) for grey cast iron FG 150, σ_{ut} = 150 N/mm².

$$\therefore \text{Allowable stress } \sigma_d = \sigma_0 = \frac{\sigma_{ut}}{FOS} = \frac{150}{5} = 30 \text{ N/mm}^2.$$

As material for cylinder is CI which is brittle material, using Lame's equation

$$\text{Thickness of cylinder wall } h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_0 + p_i}{\sigma_0 - p_i}} - 1\right] \qquad \text{----- 7.24 (DDHB)}$$

$$= \frac{150}{2}\left[\sqrt{\frac{30+5}{30-5}} - 1\right] = 13.74 \text{ mm} \cong 15 \text{ mm}$$

Allowing for blow holes and reboring, take thickness of cylinder wall h = 15 + 5 = 20 mm.

(i) **Design of head**

Consider the cover as circular plate supported at the circumference subjected to uniformly distributed load

$$\therefore \text{Thickness of head } h = k_1 d\sqrt{\frac{p}{\sigma_d}} \qquad \text{----- 8.1 (DDHB)}$$

From Table 8.3 (DDHB) for CI supported at edges. Coefficient $k_1 = 0.54$

$$\therefore h = 0.54 \times 150 \sqrt{\frac{5}{30}} = 33.068 \text{ mm}$$

From Table 8.2 (DDHB) for circular plate distributed load over the entire surface and edge supported

$$\text{Maximum allowable stress } \sigma_d = \frac{3F(3m+1)}{8\pi mh^2} \text{ where } F = \pi r_0^2 p$$

From Table 2.10 (New DDHB) for CI, Poisson's ratio $1/m = \upsilon = 0.211$

$$\text{i.e., } 30 = \frac{3 \times \pi \times 75^2 \times 5\left(3 \times \frac{1}{0.211} + 1\right)}{8\pi \times \frac{1}{0.211} \times h^2}$$

$\therefore$ Thickness of head h = 33.6 mm

Adop the bigger value, i.e., h = 33.6 mm

Take, Thickness of head h = 35 mm.

Example : 2.23

A hydraulic press shown in Fig 2.22 having a working fluid pressure of 10 N/mm². The maximum force exerted by the press is 50 kN. The allowable stress for cast steel = 100 N/mm² (Tensile), for mild steel = 50 N/mm² (compressive) and for cast iron = 30 N/mm² (tensile). Design,

(i) Ram

(ii) Cylinder

(iii) Top plate and

(iv) Sliding platform

Solution :

(i) Design of ram

Maximum force exerted by the ram $F = 50 \text{ kN} = 50 \times 10^3$ N

$$\text{i.e., } \frac{\pi}{4} d_r^2 \times p = 50 \times 10^3$$

$$\text{i.e., } \frac{\pi}{4} \times d_r^2 \times 10 = 50 \times 10^3$$

$\therefore$ Diameter of ram $d_r = 79.8 \text{ mm} \cong 80$ mm.

In order to reduce its weight, it can be designed as a thick cylinder subjected to external pressure.

According to Lame's equation, maximum tangential stress subjected to only external pressure

$$\sigma_{\theta_{max}} = -\frac{p_o d_{ro}^2}{d_{ro}^2 - d_{ri}^2}\left[1+\frac{d_{ri}^2}{d_{ro}^2}\right] \text{ when } p_i = 0,\ r = d_{ro}/2 \qquad \text{----- 7.17 (DDHB)}$$

$$= -p_o\left(\frac{d_{ro}^2 + d_{ri}^2}{d_{ro}^2 - d_{ri}^2}\right)$$

Maximum radial stress subjected to only external pressure

$$\sigma_{r_{max}} = -\frac{p_o d_{ro}^2}{d_{ro}^2 - d_{ri}^2}\left[1-\frac{d_{ri}^2}{d_{ro}^2}\right] = -p_o \qquad \text{----- 7.18 (DDHB)}$$

According to maximum shear stress theory for ductile materials, maximum shear stress

$$\tau_{max} = \frac{\sigma_{\theta_{max}} - \sigma_{r_{max}}}{2}$$

$$= \frac{-p_o\left(\frac{d_{ro}^2 + d_{ri}^2}{d_{ro}^2 - d_{ri}^2}\right) - (-p_o)}{2}$$

$$\text{i.e., } 2\tau_{max} = -p_o\left[\frac{d_{ro}^2 + d_{ri}^2 - \left(d_{ro}^2 - d_{ri}^2\right)}{d_{ro}^2 - d_{ri}^2}\right]$$

$$= -\frac{2p_o d_{ri}^2}{d_{ro}^2 - d_{ri}^2}$$

The material for the ram is mild steel. The allowable compressive stress for mild steel is given as 56 N/mm^2. As the maximum shear stress is one half the maximum allowable stress

$$2\times\frac{1}{2}\,\sigma_d = \frac{2p_o d_{ri}^2}{d_{ro}^2 - d_{ri}^2}$$

i.e., $50 = \frac{2\times 10\times d_{ri}^2}{80^2 - d_{ri}^2}$ [$\because$ Allowable stress for the ram material i.e., mild steel $\sigma_d = 50$ N/mm^2 (compressive)]

i.e., $80^2 - d_{ri}^2 = 0.4\, d_{ri}^2$

$\therefore$ Inside diameter of ram $d_{ri} = 67.61$ mm

Take, Inside diameter of ram $dr_i = 70$ mm.

(ii) Design of cylinder

Let d_o = Outside diameter of cylinder

d_i = Inside diameter of cylinder

p_i = Pressure inside the cylinder

The material for the cylinder is cast iron. It will be designed as a thick cylinder Assume a clearance of 15 mm between the ram and cylinder bore

∴ Inner diameter of cylinder $d_i = 80 + 15 = 95$ mm

According to Lame's equation, thickness of CI thick cylinder wall

$$h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta + p_i}{\sigma_\theta - p_i}} - 1\right] \qquad \text{----- 7.24 (DDHB)}$$

$$= \frac{95}{2}\left[\sqrt{\frac{30+10}{30-10}} - 1\right]$$ (∵ Maximum allowable stress for cylinder material (CI) $\sigma_0 = 30$ N/mm²)

$$= 19.67 \text{ mm} \cong 20 \text{ mm}$$

i.e., Thickness of cylinder wall h = 20 mm

∴ Outside diameter of cylinder $d_o = d_i + 2h = 95 + 2 \times 20 = 135$ mm

(iii) Design of top plate

Consider the top plate as a rectangular plate, uniformly loaded and supported at the four corners. Select the material of the top plate as cast steel.

Thickness of a rectangular plate according to Grashof's and Back

$$h = abk_3\sqrt{\frac{p}{\sigma_d\left(a^2 + b^2\right)}} \qquad \text{----- 8.7 (DDHB)}$$

where a = Length of plate = 250 mm

b = Width of plate = 250 mm

From Table 8.3 (DDHB) for steel supported ends,

Coefficient k_3 = 0.60

Pressure acting on the plate $p = \frac{50 \times 10^3}{250 \times 250} = 0.8$ N/mm²

∴ Thickness of plate $h = 250 \times 250 \times 0.6\sqrt{\frac{0.8}{100\left(250^2 + 250^2\right)}}$

$$= 9.48 \text{ mm} \cong 10 \text{ mm}$$

From Table 8.2 (DDHB) for rectangular plate, all edges supported

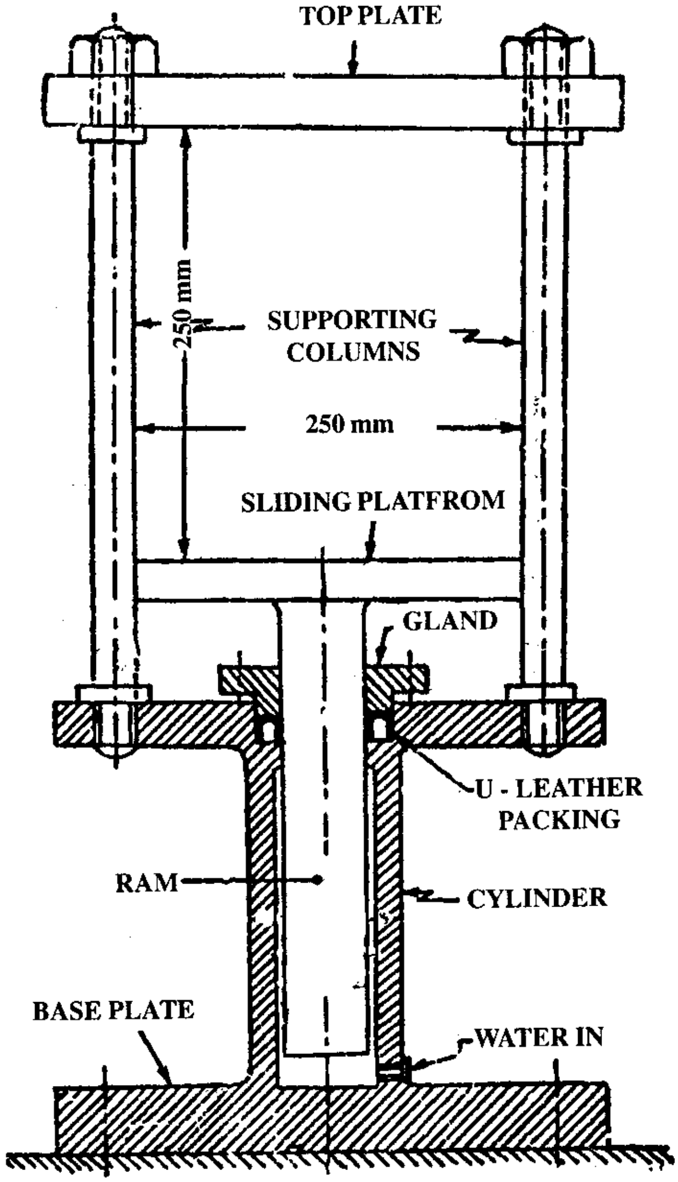


Fig 2.22 Hydraulic press

Maximum allowable stress $\sigma_d = \dfrac{0.75b^2p}{h^2\left[1+1.61\times\dfrac{b^3}{h^3}\right]}$; i.e., $100 = \dfrac{0.75\times250^2\times0.8}{h^2\left[1+1.61\times\dfrac{250^3}{250^3}\right]}$

$\therefore$ Thickness of plate h = 11.98 mm ≅ 12 mm

Adopt the larger value ; $\therefore$ Thickness of top plate h = 12 mm

(iv) Design of sliding platform

The sliding platform is a square plate, loaded uniformly on the whole face and supported at the centre on a circle of diameter of the ram. To design, this plate can be approximated by considering it as a circular plate rigidly fixed around the circumference. Consider the material of the sliding platform as mild steel.

According to Grashof's formulae for the thickness of a plate with the above given type of loading

$$h = 0.65\sqrt{\frac{F}{\sigma_d}\log_e\left(\frac{d}{d_o}\right)} \qquad \text{----- 8.5 (DDHB)}$$

where σ_d = Allowable stress = 50 N/mm²

d = Diameter of plate = 250 mm

d_o = Diameter of support = 80 mm

F = Maximum force exerted by the press = 50×10^3 N

$$\therefore h = 0.65 \times \sqrt{\frac{50\times10^3}{50}\times\log_e\left(\frac{250}{80}\right)} = 21.94 \text{ mm} \cong 25 \text{ mm}$$

∴ Thickness of sliding platform h = 25 mm.

Example : 2.24

A cast steel cylinder of 300 mm internal diameter is to contain liquid at a pressure of 12.5 N/mm². It is closed at both ends by unstayed flat cover plates and are attached by bolts. Determine the thickness of the cover plates if the allowable working stress for the cover material is 75 N/mm².

Data :

$d = 300$ mm; $p = 12.5$ N/mm²; $\sigma_d = 75$ N/mm²;

Solution :

Minimum thickness of an unstayed flat head or cover plate according to ASME Boiler code

$$h = d\sqrt{\frac{c_3 p}{\sigma_d}} \qquad \text{----- 8.40 (DDHB)}$$

From Table 8.1 (DDHB) for plates rigidly bolted to the shell flange [Fig 8.1 F (DDHB)]

Coefficient c_3 = 0.162

$$\therefore h = 300\sqrt{\frac{0.162\times12.5}{75}} = 49.3 \text{ mm} \cong 50 \text{ mm}$$

∴ Thickness of cover plate h = 50 mm